

Risk Premiums in Dynamic Term Structure Models with Unspanned Macro Risks*

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Abstract

This paper quantifies how variation in real economic activity and inflation in the U.S. influenced the market prices of level, slope, and curvature risks in U.S. Treasury markets. We develop a novel arbitrage-free dynamic term structure model in which bond investment decisions are influenced by real output and inflation risks that are *unspanned* by (imperfectly correlated with) information about the shape of the Treasury yield curve. Our model reveals that, over the period 1985–2007, these unspanned macro risks accounted for a large portion of the variation in forward terms premiums, and there was pronounced cyclical variation in the market prices of level and slope risks. We compare fitted term premiums for the post-2007 crisis period to those from a model with spanned macro risks, and use our findings to reassess some of Chairman Bernanke’s remarks on the interplay between term premiums, the shape of the yield curve, and the macroeconomy.

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1 Introduction

A powerful implication of virtually all macro-finance affine term structure models (*MTSMs*)—reduced-form and equilibrium alike—is that the macro factors that determine bond prices are fully spanned by the current yield curve.¹ That is, the affine mapping between bond yields and the risks in the macroeconomy in these models can be inverted to express these risk factors as linear combinations of yields. This theoretical macro-spanning condition implies strong and often counterfactual restrictions on the joint distribution of bond yields and the macroeconomy, as well as on how macroeconomic shocks affect term premiums.

Consider for instance an *MTSM* in which the macro variables M_t that directly determine bond yields are output growth and inflation. Macro spanning implies that these macro variables can be replicated by portfolios of bond yields. As a result, after conditioning on the current yield curve, macro variables are uninformative about *both* expected excess returns (risk premiums) and future values of M . The first of these restrictions on the joint distribution of M and bond yields is contradicted by the evidence in Cooper and Priestley (2008) and Ludvigson and Ng (2010). The second is contradicted by a large body of evidence on forecasting the business cycle (Stock and Watson (2003)). Both restrictions are strongly rejected statistically in our dataset.

There is an equally compelling conceptual case for relaxing macro spanning. The first three principal components (*PCs*) of bond yields—the level, slope, and curvature—explain almost all of the variation in yields, and this fact motivates the small number of risk factors in reduced-form *MTSMs*.² Real economic growth in the U.S. economy is a distinct agglomeration of a high-dimensional set of risks from financial, product, and labor markets. The yield *PCs* are correlated with output growth, but the natural premise in economic modeling is surely that the portfolio of risks that shape growth are not spanned by the *PCs* of U.S. Treasury yields. In fact, in our data, only about 30% of the variation in output growth is spanned by even the first five *PCs* of yields.

In this paper we develop a family of reduced-form Gaussian *MTSMs* that allows for macroeconomic risks to be unspanned by the yield curve and, thereby, introduces

¹ Reduced-form models that enforce theoretical spanning include Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2007), Rudebusch and Wu (2008), Ravenna and Seppala (2008), Smith and Taylor (2009), and Bikbov and Chernov (2010). In many equilibrium models with long-run risks (e.g., Bansal, Kiku, and Yaron (2012a), Bansal and Shaliastovich (2013)) it is expected consumption growth and expected inflation that are spanned by yields.

²See Litterman and Scheinkman (1991), Dai and Singleton (2000), and Duffee (2002) for supporting evidence. Ang, Piazzesi, and Wei (2006) and Bikbov and Chernov (2010), among others, draw explicitly on this evidence when setting the number of their risk factors.

macroeconomic risks that are distinct from PC (yield curve) risks. Central to the construction of our $MTSM$ are the assumptions that the pricing kernel that investors use when discounting cash flows depends on a comprehensive set of priced risks Z_t in the macroeconomy, and the short-term Treasury rate is an affine function of a smaller set of “portfolios” of these risks X_t (consistent with the evidence that a small number of PC s explain most of the variation in the cross section of yields). We then construct a Treasury-market specific stochastic discount factor \mathcal{M}_X such that: (i) \mathcal{M}_X prices the entire cross section of Treasury bonds; (ii) \mathcal{M}_X has market prices of X risks that may depend on the entire menu of macro risks Z ; (iii) the model-implied yields do not span Z ; and (iv) \mathcal{M}_X does not price all of these macro risks. In this manner we accommodate much richer dynamic co-dependencies among risk premiums and the macro economy than in extant $MTSM$ s.

Specializing to a setting where M_t is comprised of measures of output growth and expected inflation, we document economically large effects of the unspanned components of M_t on risk premiums in Treasury bond markets. Illustrative of our findings are the “in-two-years-for-one-year” forward term premiums $FTP_t^{2,1}$ displayed in [Figure 1](#). The premiums from our preferred model with unspanned macro risks (\mathcal{M}_{us}) show a pronounced cyclical pattern with peaks during recessions (the shaded areas), and a trough during the period Chairman Greenspan labeled the “conundrum.” Notably, there are systematic differences between $FTP_t^{2,1}$ from model \mathcal{M}_{us} and the projection of $FTP_t^{2,1}$ onto the PC s of bond yields (PM_{us}). These differences arise entirely from our accommodation of macro shocks that are unspanned by yields. Unspanned macro risks have their largest impacts on $FTP_t^{2,1}$ during the peaks and troughs of business cycles, as well as during the conundrum period.

Enforcing macro spanning within a $MTSM$ (constraining \mathcal{M}_{us} and PM_{us} to be identical) can lead to highly inaccurate model-implied risk premiums. Consider, for instance, the fitted $FTP_t^{2,1}$ (\mathcal{M}_{span}) from the $MTSM$ that (incorrectly) constrains expected output growth and inflation to be spanned by the yield PC s. Both PM_{us} and \mathcal{M}_{span} are exact linear combinations of yield PC s. Yet their differences are often huge, with \mathcal{M}_{span} often declining when PM_{us} is increasing. We subsequently use these implied premiums to reassess recent interpretations of the interplay between term premiums, the shape of the yield curve, and macroeconomic activity, including those of Chairman Bernanke.³

While the extant literature is vast, we are unaware of prior research that explores the relationship between unspanned macro shocks and risk premiums in bond markets

³See, for example, his speech before the Economic Club of New York on March 20, 2006 titled “Reflections on the Yield Curve and Monetary Policy.” His talks draw explicitly on the model estimated by Kim and Wright (2005), and their model is nested in our canonical model.

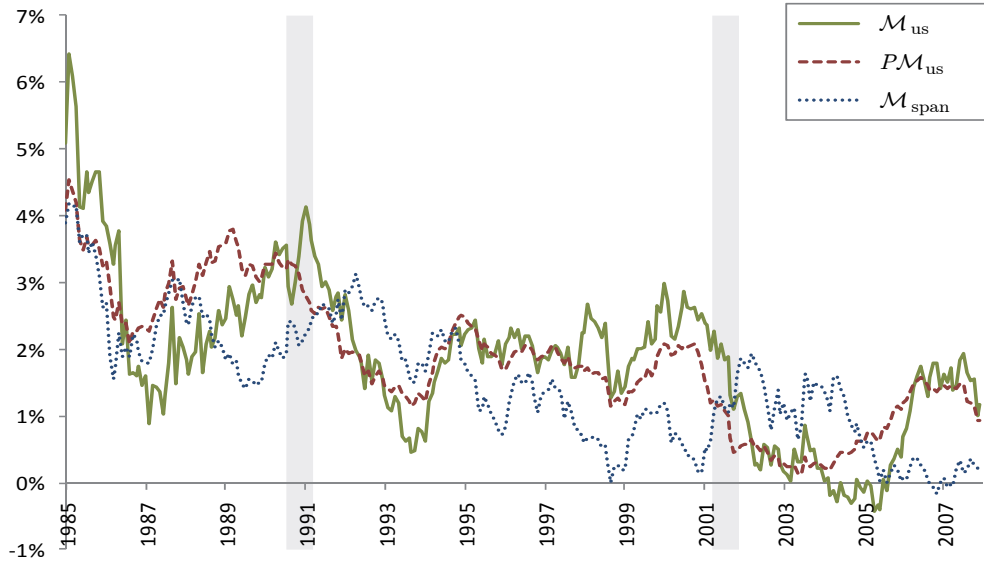


Figure 1: The “in-two-years-for-one-year” forward term premiums $FTP_t^{2,1}$, defined as the difference between the expectation for two years in the future of the one-year yield, and the forward rate that one could lock in today for a one-year loan commencing in two years. We plot $FTP_t^{2,1}$ implied by our preferred model with unspanned macro risks (\mathcal{M}_{us}), the projection of $FTP_t^{2,1}$ from model \mathcal{M}_{us} onto the first three PC s of bond yields ($P\mathcal{M}_{us}$), and the $FTP_t^{2,1}$ implied by the nested model that enforces spanning of expectations of the macro variables by the yield PC s (\mathcal{M}_{span}).

within arbitrage-free pricing models. Independently, Duffee (2011) proposes a latent factor (yields-only) model for accommodating unspanned risks in bond markets.⁴ We formally derive a canonical form for $MTSM$ s with unspanned information that affects expected excess returns, and provide a convenient normalization that ensures econometric identification. Moreover, as we illustrate, the global optimum of the associated likelihood function is achieved extremely quickly. Wright (2011) and Barillas (2011) use our framework to explore the effects of inflation uncertainty on bond market risk premiums using international data, and optimal bond portfolio choice in the presence of macro-dependent market prices of risk, respectively.

The remainder of this paper is organized as follows. [Section 2](#) reviews the modeling choices made in the current generation of $MTSM$ s, and argues that these models

⁴Duffee (2011) does not explore the econometric identification of such a model, nor does he empirically implement a dynamic term structure model with unspanned risks.

enforce strong and counterfactual restrictions on how the macroeconomy affects yields. [Section 3](#) then proposes a canonical *MTSM* with unspanned macro risks that takes a large step towards bringing *MTSMs* in line with the historical evidence. The associated likelihood function is derived in [Section 4](#). Our formal estimation and the model-implied risk premiums on exposures to “level” and “slope” risks are presented in [Section 5](#). The properties of risks premiums in our *MTSM* are explored in more depth in [Section 6](#) by examining the links between macroeconomic shocks and the time-series properties of forward term premiums. There we elaborate on [Figure 1](#), as well as counterparts for longer-dated forward term premiums. In [Section 7](#), we document that unspanned macro risks had economically significant effects on the shape of the forward premium curve. [Section 8](#) elaborates on the structure of our *MTSM* and explores the robustness of our empirical findings to extending our sample well into the current crisis period. In [Section 9](#), we consider several extensions. Finally, [Section 10](#) concludes.

2 Empirical Observations Motivating Our *MTSM*

Consider an economic environment in which agents value nominal bonds using the stochastic discount factor

$$\mathcal{M}_{Z,t+1} = e^{-r_t - \frac{1}{2}\Lambda'_{Zt}\Lambda_{Zt} - \Lambda'_{Zt}\eta_{t+1}^{\mathbb{P}}}; \quad (1)$$

the $R \times 1$ state-vector Z_t encompasses all risks in the economy. Suppose that Z_t follows the Gaussian process⁵

$$Z_t = K_{0Z}^{\mathbb{P}} + K_{1Z}^{\mathbb{P}}Z_{t-1} + \sqrt{\Sigma_Z}\eta_t^{\mathbb{P}} \quad \eta_t^{\mathbb{P}} \sim N(0, I); \quad (2)$$

the market prices Λ_{Zt} of the risks $\eta_{t+1}^{\mathbb{P}}$ are affine functions of Z_t ; and the yield on a one-period bond r_t is an affine function of Z_t ,

$$r_t = \rho_{0Z} + \rho_{1Z} \cdot Z_t. \quad (3)$$

Bond prices are then computed with standard recursions; see [Appendix A](#). This formulation encompasses virtually all of the Gaussian *MTSMs* in the literature.

Perhaps the most salient feature of these *MTSMs* is that Z_t includes a set of macro risk factors M_t , typically measures of output growth and inflation (for examples,

⁵Our analysis easily extends to the case where (2) is the companion form of a high-order vector-autoregressive (*VAR*) representation of Z . Below we provide empirical evidence supporting our assumption that Z follows a first-order *VAR* with nonsingular Σ_Z .

see the references in [footnote 1](#)). Joslin, Le, and Singleton (2013) (JLS) show that, for such choices of Z_t , except in degenerate cases, (1)-(3) is theoretically equivalent to a *MTSM* in which Z_t is *normalized* to the first R *PCs* of bond yields, denoted by \mathcal{P} , so that

$$r_t = \rho_{0\mathcal{P}} + \rho_{1\mathcal{P}} \cdot \mathcal{P}_t; \quad (4)$$

and M_t is related to \mathcal{P}_t through the macro-spanning restriction

$$M_t = \gamma_0 + \gamma_{1\mathcal{P}} \cdot \mathcal{P}_t. \quad (5)$$

Thus, the only feature of extant *MTSMs* that differentiates them from term structure models with no macro risk factors and r_t specified as in (4) (Duffee (2002), Joslin, Singleton, and Zhu (2011) (JSZ)) is the restriction (5) that M_t is spanned by \mathcal{P}_t .

To motivate the specification of our canonical *MTSM*, we highlight the three observations that challenge the empirical plausibility of this family of *MTSMs*. First, output, inflation, and other macroeconomic risks are not linearly spanned by the information in the yield curve. Second, the unspanned components of many macro risks have predictive power for excess returns (risk premiums) in bond markets, over and above the information in the yield curve. Third, the cross section of bond yields is well described by a low-dimensional set of risk factors.

Macroeconomic Risks Are Unspanned by Bond Yields

For our subsequent empirical analysis, we include measures of real economic activity (*GRO*) and inflation (*INF*) in M_t . *GRO* is measured by the three-month moving average of the Chicago Fed National Activity Index, a measure of current real economic conditions.⁶ *INF* is measured as the expected rate of inflation over the coming year as computed from surveys of professional forecasters by Blue Chip Financial Forecasts.⁷ We make the parsimonious choice of $M'_t = (GRO_t, INF_t)$ as these risks have received the most attention in prior studies.⁸

⁶ The Federal Reserve Bank of Chicago constructs the CFNAI from economic indicators from the categories: production and income (23 series), employment and hours (24 series), personal consumption and housing (15 series), and sales, orders, and inventories (23 series). The data is inflation adjusted. The methodology used is similar to that used by Stock and Watson (1999) to construct their index of real economic activity, and it is also related to the *PCs* of economic activity used by Ludvigson and Ng (2010) to forecast excess returns in bond markets.

⁷The CFNAI for a specific month is first published during the following calendar month, and subject to revisions. The Blue Chip forecasts are available in real time subject only to at most a few days' lag.

⁸Ang et al. (2006) and Jardet, Monfort, and Pegoraro (2011) focus on models with GRO_t being the sole macro risk. Kim and Wright (2005) explore *MTSMs* with expected inflation being the sole macro risk. Bikbov and Chernov (2010) and Chernov and Mueller (2012) examine models with

As evidence on the macro-spanning condition (5), consider the projection of GRO and INF onto the PC s of yields on U.S. Treasury nominal zero-coupon bonds with maturities of six months and one through five, seven, and ten years.⁹ The projection of GRO onto the first three PC s gives an (adjusted) R^2 of 15%, so about 85% of the variation in GRO arises from risks distinct from $\mathcal{P}_t^{3'} = (PC1, PC2, PC3)$. Adding $PC4$ and $PC5$ as regressors only raises the R^2 for GRO to 32%. The comparable R^2 's for INF are 83% (\mathcal{P}^3) and 86% (\mathcal{P}^5).

Macro Risk Factors Forecast Bond Excess Returns

Not only is M_t unspanned by \mathcal{P}_t^3 , but the projection error $OM_t = M_t - \text{Proj}[M_t | \mathcal{P}_t^3]$ has considerable predictive power for excess returns, over and above \mathcal{P}^3 . For instance, consider the one-year holding period returns on two-year and ten-year bonds, xr_{t+12}^2 and xr_{t+12}^{10} . The adjusted R^2 from the projection of xr_{t+12}^2 (xr_{t+12}^{10}) onto \mathcal{P}_t^3 is 0.14 (0.20), while onto $\{\mathcal{P}_t^3, GRO_t, INF_t\}$ it is 0.48 (0.37).¹⁰ If we project the excess returns onto \mathcal{P}_t^5 , the adjusted R^2 drop to 0.27 and 0.22.¹¹

Bond Yields Follow a Low-Dimensional Factor Model

Another salient feature of the yield curves in most developed countries is that the cross section of bond yields is well described by a low-dimensional factor model. Often three or four factors explain nearly all of the cross-sectional variation in yields.

These empirical observations highlight an inherent tension in $MTSM$ s that enforce versions of the spanning condition (5), one that likely compromises their goodness-of-fits and the reliability of their inferences about the dynamic relationships between macro risks and the yield curve. In particular, JLS show that, for the typical case of $R = 3$ and $M'_t = (GRO_t, INF_t)$ measured perfectly, canonical $MTSM$ s fit individual yields poorly, with pricing errors exceeding 100 basis points in some periods. Furthermore, adding measurement errors on M_t leads the likelihood function to effectively drive out the macro factors leaving filtered risk factors that more closely

$M'_t = (GRO_t, INF_t)$. Only Chernov and Mueller (2012) relax the macro-spanning constraint by allowing expected inflation to be unspanned by real yields; our framework is substantially more general in that we allow arbitrary factors to be unspanned by either the real or nominal yield curve.

⁹The zero curves for U.S. Treasury series are described in more depth in Le and Singleton (2013). The zero curves are constructed using the same bond selection criteria as in the Fama-Bliss data used in many previous studies. Importantly, we are using a consistent series out to ten years to maturity, and throughout our sample period.

¹⁰The descriptive analysis in Cieslak and Povala (2010) provides complementary evidence that the unspanned component of inflation has substantial predictive content for excess returns in bond markets. Our modeling framework allows the accommodation of their findings within a $MTSM$.

¹¹If we restrict our sample to end in 2003, as in Cochrane and Piazzesi (2005), the adjusted R^2 for projecting xr_{t+12}^2 and xr_{t+12}^{10} onto \mathcal{P}_t^5 are 0.28 and 0.30, respectively

resemble \mathcal{P}_t^3 . In light of this evidence, it seems doubtful that low-dimensional factor models in which macro variables comprise half or more of the risk factors provide reliable descriptions of the joint dynamics of macro and yield curve risks.

Expanding the number of risk factors (increasing R) mitigates the fitting problem for bond yields, but at the expense of over-parameterizing the risk-neutral distribution of Z_t . The consequent over-fitting of *MTSMs* is material: both Duffee (2010) and JSZ document that model-implied Sharpe ratios for certain bond portfolios are implausibly large when R is as low as four. This problem is likely to be exacerbated in *MTSMs*, since an even larger R (relative to yields-only models) may be needed to accurately price individual bonds.

We overcome these problems by specifying a canonical *MTSM* with the following fitting properties:

FP1: the number of risk factors is small (three in our empirical implementation);

FP2: the macroeconomic risks are unspanned by bond yields; and

FP3: the unspanned components of M_t have predictive content for excess returns.

We show that all of these features arise naturally from the projection of agents' economy-wide pricing kernel onto the set of risk factors that characterize the cross-sectional distribution of Treasury yields. That is, taking as given the low-dimensional factor structure of bond yields FP1, features FP2 and FP3 are direct consequences of agents' attitudes towards risks in the broader economy.

3 A Canonical Model with Unspanned Macro Risks

Consistent with FP1, suppose that a low-dimensional R -vector of portfolios of risks determines the one-period bond yield r_t according to (4). At the same time, let us generalize the generic pricing kernel (1)-(2) to the one capturing the $N > R$ economy-wide risks Z_t underlying all tradable assets available to agents in the economy. Conceptually, the dimension reduction from N to R (from Z to \mathcal{P}) in (4), implied by FP1, could arise because the economy-wide risks underlying \mathcal{M}_Z impinge on bond yields only through the R portfolios of risks \mathcal{P} . Alternatively, $N > R$ could arise because certain risks in $\eta_t^{\mathbb{P}}$ (e.g., cashflow risks in equity markets) are largely inconsequential for the pricing of Treasury bonds. In either case, \mathcal{P}_t and Z_t will in general be correlated, but Z_t will not be deterministically related to \mathcal{P}_t .

Most *MTSMs* are designed to price zero-coupon bonds in a specific bond market¹² and, as such, their pricing kernels are naturally interpreted as projections of the economy-wide \mathcal{M}_Z onto the portfolios of risks \mathcal{P}_t that specifically underlie variation in bond yields. Pursuing this logic in a notationally parsimonious way, we suppose that the macro risks of interest M_t “complete” the state vector in the sense that (\mathcal{P}'_t, M'_t) and Z_t represent linear rotations of the same N risks.¹³ Then, to construct our bond-market specific $\mathcal{M}_{\mathcal{P},t+1}$, we project $\mathcal{M}_{Z,t+1}$ onto \mathcal{P}_{t+1} (the priced risks in the bond market) and Z_t (the state of the economy) to obtain:

$$\mathcal{M}_{\mathcal{P},t+1} \equiv \text{Proj} [\mathcal{M}_{Z,t+1} | \mathcal{P}_{t+1}, Z_t] = e^{-r_t - \frac{1}{2} \Lambda'_{\mathcal{P}t} \Lambda_{\mathcal{P}t} - \Lambda'_{\mathcal{P}t} \epsilon_{\mathcal{P},t+1}^{\mathbb{P}}}. \quad (6)$$

Though (6) resembles the kernels in previous studies with spanned macro risks, there are several crucial differences. The risks $\epsilon_{\mathcal{P},t+1}^{\mathbb{P}}$ in (6) are the first R innovations from the unconstrained *VAR*

$$\begin{bmatrix} \mathcal{P}_t \\ M_t \end{bmatrix} = \begin{bmatrix} K_{0\mathcal{P}}^{\mathbb{P}} \\ K_{0M}^{\mathbb{P}} \end{bmatrix} + \begin{bmatrix} K_{\mathcal{P}\mathcal{P}}^{\mathbb{P}} & K_{\mathcal{P}M}^{\mathbb{P}} \\ K_{M\mathcal{P}}^{\mathbb{P}} & K_{MM}^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \mathcal{P}_{t-1} \\ M_{t-1} \end{bmatrix} + \sqrt{\Sigma_Z} \epsilon_{Zt}^{\mathbb{P}}, \quad (7)$$

where $\epsilon_{Zt}^{\mathbb{P}} \sim N(0, I_N)$, the $N \times N$ matrix Σ_Z is nonsingular, and $\Sigma_{\mathcal{P}\mathcal{P}}$ is the upper $R \times R$ block of Σ_Z . Accordingly, consistent with features FP2 and FP3, M_t is not deterministically spanned by \mathcal{P}_t and forecasts of \mathcal{P} are conditioned on the full set of N risk factors Z_t .¹⁴

To close our model, we assume that \mathcal{P}_t follows an autonomous Gaussian *VAR* under the pricing (risk-neutral) distribution \mathbb{Q} :

$$\mathcal{P}_t = K_{0\mathcal{P}}^{\mathbb{Q}} + K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}} \mathcal{P}_{t-1} + \sqrt{\Sigma_{\mathcal{P}\mathcal{P}}} \epsilon_{\mathcal{P}t}^{\mathbb{Q}}. \quad (8)$$

Under these assumptions and the absence of arbitrage opportunities, the yield on an m -period bond, for any $m > 0$, is an affine function of \mathcal{P}_t ,

$$y_t^m = A_{\mathcal{P}}(m) + B_{\mathcal{P}}(m) \cdot \mathcal{P}_t, \quad (9)$$

¹²Two exceptions are the reduced-form equity and bond pricing models studied by Lettau and Wachter (2011) and Kojien, Lustig, and van Nieuwerburgh (2012). These models raise spanning issues as well. For instance, the Kojien et al. (2012) model implies that the value-weighted return on the *NYSE* is a linear combination of three *PCs* of bond yields.

¹³Our key points are easily derived for the case where Z_t includes more risks than those spanned by (\mathcal{P}'_t, M'_t) . Also, implicit in our construction is the assumption that $N - R$ elements of M_t are unspanned by the yield portfolios \mathcal{P}_t .

¹⁴In this respect, (7) is very similar to the descriptive six-factor model studied by Diebold, Rudebusch, and Aruoba (2006). As in their analysis, we emphasize the joint determination of the macro and yield variables. We overlay a no-arbitrage pricing model with unspanned macro risk in order to explore their impact on risk premiums in bond markets.

where the loadings $A_{\mathcal{P}}(m)$ and $B_{\mathcal{P}}(m)$ are known functions of the parameters governing the \mathbb{Q} distribution of yields (see [Appendix A](#)). Without loss of generality, we rotate the risk factors so that \mathcal{P} corresponds to the first R PCs of yields.¹⁵

The market prices of risk in [\(6\)](#),

$$\Lambda_{\mathcal{P}}(Z_t) = \Sigma_{\mathcal{P}\mathcal{P}}^{-1/2} (\mu_{\mathcal{P}}^{\mathbb{P}}(Z_t) - \mu_{\mathcal{P}}^{\mathbb{Q}}(\mathcal{P}_t)), \quad (10)$$

are constructed from the drift of \mathcal{P}_t under \mathbb{P} (obtained from [\(7\)](#)) and the drift of \mathcal{P}_t under \mathbb{Q} (obtained from [\(8\)](#)). They are affine functions of Z_t , even though the only (potentially) priced risks in Treasury markets are \mathcal{P}_t . Thus, agents' risk tolerance is influenced by information broadly about the state of the economy. It follows that agents' pricing kernel cannot be represented in terms of \mathcal{P}_t alone. Furthermore, our framework implies that the residual OM_t in the linear projection

$$M_t = \gamma_0 + \gamma_{1\mathcal{P}} \cdot \mathcal{P}_t + OM_t \quad (11)$$

is informative about the primitive shocks impinging on the macroeconomy and, therefore, about risk premiums and future bond yields.

In contrast, the spanning condition [\(5\)](#) (i.e., supposing that the unspanned macro risks OM_t in [\(11\)](#) is identically zero) adopted by the vast majority of *MTSMs* presumes that OM_t is identically zero. Economic environments that maintain this constraint have the property that all aggregate risk impinging on the future shape of the yield curve can be fully summarized by the yield PCs \mathcal{P}_t . In particular, the spanning condition implies that the past history of M_t is irrelevant for forecasting not only future yields, but also future values of M , once one has conditioned on \mathcal{P}_t . It follows that *MTSMs* that enforce spanning fail to satisfy fitting properties FP2 and FP3.

Not only might there be important effects of OM_t on expected excess returns, but the market prices of *spanned* macro risks may well be affected by OM_t . In particular, the market price of spanned inflation risk, an easily computable linear combination of the market prices of the *PC* risks $\Lambda_{\mathcal{P}}(Z_t)$, may be very different from its counterpart in a model that assumes inflation risks are spanned by *PCs*.¹⁶

¹⁵This rotation is normalized so that the parameters governing the \mathbb{Q} distribution of yields— $(\rho_0, \rho_{\mathcal{P}}, K_{0\mathcal{P}}^{\mathbb{Q}}, K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}})$ —are fully determined by the parameter set $(\Sigma_{\mathcal{P}\mathcal{P}}, \lambda^{\mathbb{Q}}, r_{\infty}^{\mathbb{Q}})$ (see JSZ), where $\lambda^{\mathbb{Q}}$ denotes the R -vector of ordered non-zero eigenvalues of $K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}}$ and $r_{\infty}^{\mathbb{Q}}$ denotes the long-run mean of r_t under \mathbb{Q} . As in JSZ, we can accommodate repeated and complex eigenvalues. As they show, a minor modification allows us to consider zero eigenvalues in the canonical form. $(\lambda^{\mathbb{Q}}, r_{\infty}^{\mathbb{Q}})$ are rotation invariant (that is, independent of the choice of pricing factors) and, hence, are economically interpretable parameters.

¹⁶A generic feature of all reduced-form *MTSMs* designed to price nominal Treasury bonds is

We stress that whether or not a macro-*DTSM* embodies the spanning property (5) is wholly independent of the issue of errors in measuring either bond yields or macro factors. As typically parameterized in the literature, measurement errors are independent of economic agents' decision problems and, hence, of the economic mechanisms that determine bond prices.

Interestingly, the framework of Kim and Wright (2005), the model cited by Chairman Bernanke when discussing the impact of the macroeconomy on bond market risk premiums, formally breaks the perfect spanning condition (5), but without incorporating FP3. Kim and Wright assume that M_t is inflation, and they arrive at their version of (11) by assuming that *expected* inflation is spanned by the pricing factors in the bond market. They additionally assume that \mathcal{P} follows an autonomous Gaussian process under \mathbb{Q} so their model and ours imply exactly the same bond prices. However, the \mathbb{P} -distribution of Z_t implied by their assumptions (adapted to our framework) is:

$$\begin{bmatrix} \mathcal{P}_t \\ M_t \end{bmatrix} = \begin{bmatrix} K_{0\mathcal{P}}^{\mathbb{P}} \\ \gamma_0 \end{bmatrix} + \begin{bmatrix} K_{\mathcal{P}\mathcal{P}}^{\mathbb{P}} & 0 \\ \gamma'_{1\mathcal{P}} K_{\mathcal{P}\mathcal{P}}^{\mathbb{P}} & 0 \end{bmatrix} \begin{bmatrix} \mathcal{P}_{t-1} \\ M_{t-1} \end{bmatrix} + \sqrt{\Sigma_Z} \begin{bmatrix} \epsilon_{\mathcal{P}t}^{\mathbb{P}} \\ \eta_t \end{bmatrix}, \quad (12)$$

where $\eta_t = (\nu_t + \gamma'_{1\mathcal{P}} \sqrt{\Sigma_{\mathcal{P}\mathcal{P}}} \epsilon_{\mathcal{P}t}^{\mathbb{P}})$. Thus, the Kim-Wright formulation leads to a constrained special case of our model under which the history of M_t has no forecasting power for future values of M or \mathcal{P} , once one conditions on the history of \mathcal{P} . As we will see, the zero restrictions in (12) are strongly rejected in our data.

Left open by this discussion is the issue of whether our model is canonical in the sense that all R -factor *MTSMs* with $N - R$ unspanned macro risks are observationally equivalent to a model in the class we specify here. We show in [Appendix B](#) the conditions on the latent factor model to allow for unspanned risks. We then show in [Appendix C](#) that every model with unspanned macro risk is observationally equivalent to our *MTSM* with the state vector $Z'_t = (\mathcal{P}'_t, M'_t)$, where \mathcal{P}_t are the first R principal components of y_t .¹⁷

that one cannot identify the market prices of the full complement of risks Z_t from the bond-market specific pricing kernel \mathcal{M}_X . This means, in particular, that the market prices of the *total*—spanned plus unspanned—macro risks are not econometrically identified, because nominal bond prices are not sensitive to the risk premiums that investors demand for bearing the unspanned macro risks. The market prices of unspanned inflation risk are potentially identified from TIPS yields, as in D'Amico, Kim, and Wei (2008) and Campbell, Sunderam, and Viceira (2013). However, the introduction of TIPS raises new issues related to illiquidity and data availability, so we follow most of the extant literature and focus on nominal bond yields alone.

¹⁷[Appendix C](#) also gives the explicit construction of $(\rho_0, \rho_{\mathcal{P}}, K_{0\mathcal{P}}^{\mathbb{Q}}, K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}})$ from $(\Sigma_{\mathcal{P}\mathcal{P}}, \lambda^{\mathbb{Q}}, r_{\infty}^{\mathbb{Q}})$ for our choice of \mathcal{P} as a vector of yield *PCs*.

4 The Likelihood Function

In constructing the likelihood function for our canonical *MTSM* we let y_t denote the J -dimensional vector of bond yields ($J > N$) to be used in assessing the fit of an *MTSM*. We assume that Z_t , including \mathcal{P}_t , is measured without error and that the remaining $J - R$ *PCs* of the yields y_t , $PC^{el} \equiv (PC(R+1), \dots, PCJ)$, are priced with *i.i.d.* $N(0, \Sigma_e)$ errors. Sufficient conditions for any errors in measuring (pricing) \mathcal{P}_t to be inconsequential for our analysis are derived in JLS, and experience shows that the observed low-order *PCs* comprising \mathcal{P}_t are virtually identical to their filtered counterparts in models that accommodate errors in all *PCs*. With this error structure, the conditional density of (Z_t, PC_t^e) is:

$$\begin{aligned} f(Z_t, PC_t^e | Z_{t-1}; \Theta) &= f(PC_t^e | Z_t, Z_{t-1}; \Theta) \times f(Z_t | Z_{t-1}; \Theta) \\ &= f(PC_t^e | PC_t; \lambda^{\mathbb{Q}}, r_{\infty}^{\mathbb{Q}}, L_Z, L_e) \times f(Z_t | Z_{t-1}; K_Z^{\mathbb{P}}, K_0^{\mathbb{P}}, L_Z), \end{aligned} \quad (13)$$

where L_Z and L_e are the Cholesky factorizations of Σ_Z and Σ_e , respectively.

A notable property of the log-likelihood function associated with (13) is the complete separation of the parameters $(K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}})$ governing the conditional mean of the risk factors from those governing risk-neutral pricing of the bond yields and *PCs*. Absent further restrictions, the *ML* estimators of $(K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}})$ are recovered by standard linear projection.

Even more striking is the implication of (13) that the least-squares estimators of $(K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}})$ are invariant to the imposition of restrictions on the \mathbb{Q} distribution of (Z_t, y_t) . In particular, consider the following two canonical *MTSMs* with identical state vector $Z_t' = (\mathcal{P}_t', M_t')$: model 1 has $R < N$ pricing factors normalized to \mathcal{P}_t , and model 2 has N pricing factors normalized to Z_t . Model 1 is precisely our *MTSM*. In contrast, model 2 is equivalent to a *MTSM* in which the pricing factors are the first N *PCs* of yields and the spanning condition (5) is enforced. In both of these models the likelihood function factors as in (13) and, therefore, *both models imply identical ML estimates $(K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}})$ and, hence, identical optimal forecasts of Z .*

Pursuing this comparison, the implausibly large Sharpe ratios that arise in models of type 2 with relatively large N must arise from over-fitting the pricing distribution of the risk factors, $f(PC_t^e | Z_t, Z_{t-1})$. We avoid this over-fitting by adopting a more parsimonious $f(PC_t^e | Z_t, Z_{t-1})$ (shrinking N factors down to R).¹⁸ The pricing kernel

¹⁸Certainly other sets of constraints on an N -factor pricing model might avoid the over-specification of $f(PC_t^e | Z_t, Z_{t-1})$. However, care must be exercised in choosing these constraints so as to avoid solving a problem with the \mathbb{Q} distribution at the expense of contaminating the \mathbb{P} distribution of Z . The possibility of transferring mis-specification from the \mathbb{Q} to the \mathbb{P} distribution arises, for example, when constraints are imposed on $\Lambda_{\mathcal{P}}(Z_t)$ to attenuate excessive Sharpe ratios (Duffee (2010)).

underlying our *MTSM* has the appealing interpretation as the projection of agents' kernel onto the factors \mathcal{P}_t that, consistent with FP1, describe the cross section of bond yields. Moreover, this parsimony is achieved with the likelihood function of our canonical *MTSM* being fully unencumbered in fitting the conditional mean of Z_t , thereby offering maximal flexibility in matching FP2 and FP3.

5 Risk Premium Accounting

Our sample extends from January, 1985 through December, 2007. There is substantial evidence that the Federal Reserve changed its policy rule during the early 1980's, following a significant policy experiment (Clarida, Gali, and Gertler (2000), Taylor (1999), and Woodford (2003)). Our starting date is well after the implementation of new operating procedures, and covers the Greenspan and early Bernanke regimes. See Section 8 for a discussion of alternative sample periods. Consistent with the literature, we tie the choice of the number of risk factors underlying bond prices (R) to the cross-sectional factor structure of yields over the range of maturities we examine. Over 99% of the variation in yields is explained by their first three *PCs*, so we set $R = 3$ and, without loss of generality (see Section 3), normalize \mathcal{P}_t to be these three *PCs*. M_t includes the measures of output growth and expected inflation (*GRO*, *INF*) described in Section 2, so that $N = 5$. The time series ($\mathcal{P}'_t, GRO_t, INF_t$) are displayed in Figure 2.¹⁹

With ($R = 3, N = 5$) our canonical model with unspanned macro risk has forty-five parameters governing the \mathbb{P} distribution of Z (those comprising $K_0^{\mathbb{P}}, K_Z^{\mathbb{P}}$, and L_Z). There are four additional parameters governing the \mathbb{Q} distribution of Z ($r_{\infty}^{\mathbb{Q}}$ and $\lambda^{\mathbb{Q}}$). Faced with such a large number of free parameters, we proceed with a systematic model-selection search over admissible parameterizations of the market prices of \mathcal{P} risks. The scaled market prices of risk, $\Sigma_{\mathcal{P}\mathcal{P}}^{1/2}\Lambda_{\mathcal{P}}(Z_t)$, depend on the fifteen parameters of the matrix $\Lambda_1 \equiv K_{\mathcal{P}Z}^{\mathbb{P}} - [K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}} \ 0_{3 \times 2}]$ governing state-dependence, where $K_{\mathcal{P}Z}^{\mathbb{P}}$ is the first three rows of $K_Z^{\mathbb{P}}$, and also on the three intercept terms $\Lambda_0 \equiv K_{0\mathcal{P}}^{\mathbb{P}} - K_{0\mathcal{P}}^{\mathbb{Q}}$. We address two distinct aspects of model specification with our selection exercise.

First, we seek the *best* set of zero restrictions on these eighteen parameters governing risk premiums, trading off good fit against over-parameterization. Exploiting the structure of our *MTSM*, we show in Appendix D that, to a first-order approximation, the first row of $\Lambda_0 + \Lambda_1 Z_t$ is the excess return on the yield portfolio whose value

¹⁹Letting $\ell_{j,i}$ denote the loading on *PCj* in the decomposition of yield *i*, the *PCs* have been rescaled so that (1) $\sum_{i=1}^8 \ell_{1,i}/8 = 1$, (2) $\ell_{2,10y} - \ell_{2,6m} = 1$, and (3) $\ell_{3,10y} - 2\ell_{3,2y} + \ell_{3,6m} = 1$. This puts all the *PCs* on similar scales.

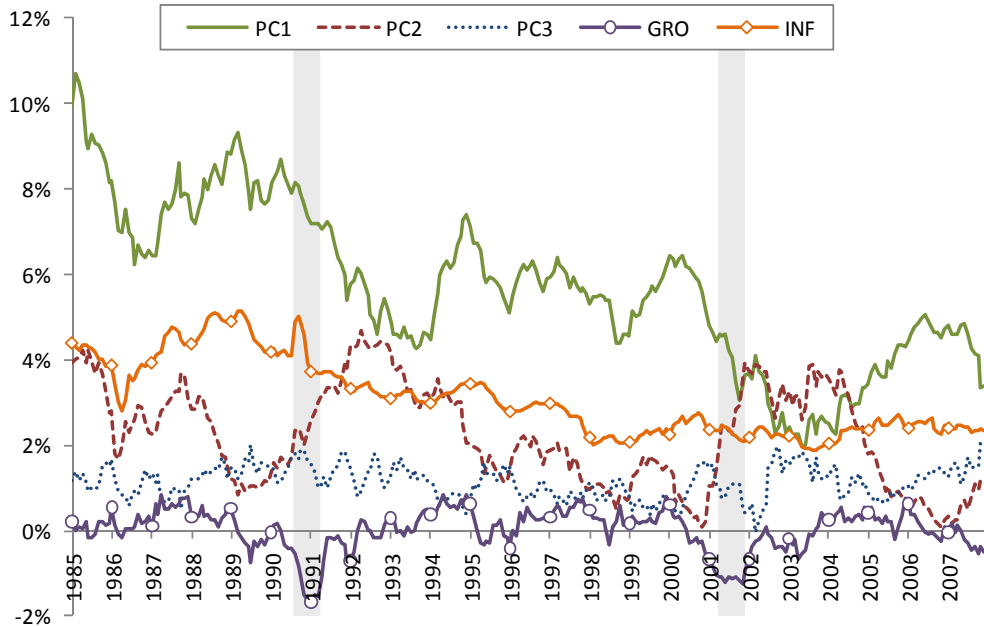


Figure 2: **Term Structure and Macro Variables** This figure plots the time series of $(PC1, PC2, PC3)$ of U.S. Treasury-implied zero yields as well as macro variables GRO and INF . The shaded areas mark NBER recessions. GRO is measured by the three-month moving average of the Chicago Fed National Activity Index and INF is measured as the expected rate of inflation over the coming year as computed from surveys of professional forecasters by Blue Chip Financial Forecasts.

changes (locally) one-for-one with changes in $PC1$, but whose value is unresponsive to changes in $PC2$ or $PC3$. Similar interpretations apply to the second and third rows of $\Lambda_0 + \Lambda_1 Z_t$, for $PC2$ and $PC3$. By examining the behavior of the expected excess returns on these PC -mimicking portfolios, xPC_{jt} ($j = 1, 2, 3$), we gain a new perspective on the nature of priced risks in Treasury markets. This economic interpretation of the constraints on $[\Lambda_0 \ \Lambda_1]$ is a benefit of our canonical form; no such model-free interpretation is possible within a latent factor model.

Second, in applying these selection criteria, we are mindful of the near unit-root behavior of yields under both \mathbb{P} and \mathbb{Q} . There is substantial evidence that bond yields are nearly cointegrated (e.g., Giese (2008), Jardet, Monfort, and Pegoraro (2011)). We also find that $PC1$, $PC2$, and INF exhibit behavior consistent with a near cointegrating relationship, whereas $PC3$ and GRO appear stationary. While

we do not believe that (*PC1*, *PC2*, *INF*) literally embody unit-root components, it may well be beneficial to enforce a high degree of persistence under \mathbb{P} , since *ML* estimators of drift parameters are known to be biased in small samples (Yamamoto and Kunitomo (1984)). This bias tends to be proportionately larger the closer a process is to a unit root process (Phillips and Yu (2005), Tang and Chen (2009)).

Moreover, when $K_Z^{\mathbb{P}}$ is estimated from a *VAR*, its largest eigenvalue tends to be sufficiently below unity to imply that expected future interest rates out ten years or longer are virtually constant (see below). This is inconsistent with surveys on interest rate forecasts (Kim and Orphanides (2005)),²⁰ and leads to the attribution of too much of the variation in forward rates to variation in risk premiums.

To address this persistence bias, we exploit two robust features of *MTSMs*: the largest eigenvalue of $K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}}$ tends to be close to unity, and the cross section of bond yields precisely identifies the parameters of the \mathbb{Q} distribution (in our case, $r_{\infty}^{\mathbb{Q}}$ and $\lambda^{\mathbb{Q}}$). Any zero restrictions on Λ_1 called for by our model selection criteria effectively pull $K_Z^{\mathbb{P}}$ closer to $K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}}$, so the former may inherit more of the high degree of persistence inherent in the latter matrix. In addition, we call upon our model selection criteria to evaluate whether setting the largest eigenvalues of the feedback matrices $K_Z^{\mathbb{P}}$ and $K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}}$ equal to each other improves the quality of our *MTSM*. Through both channels we are effectively examining whether the high degree of precision with which the cross section of yields pins down $\lambda^{\mathbb{Q}}$ is reliably informative about the degree of persistence in the data-generating process for Z_t . Again, this exploration is not possible absent the structure of a *MTSM*.²¹

5.1 Selecting Among Models

Since there are eighteen free parameters governing risk premiums, there are 2^{18} possible configurations of *MTSMs* with some of the risk-premium parameters set to zero. We examine each of these models with and without the eigenvalue constraint across $K_Z^{\mathbb{P}}$ and $K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}}$, for a total of 2^{19} specifications. Though 2^{19} is large, the rapid convergence to the global optimum of the likelihood function obtained using our normalization scheme makes it feasible to undertake this search using formal model selection criteria. For each of the 2^{19} specifications examined, we compute full-information *ML* estimates of the parameters and then evaluate the Akaike (1973)

²⁰Similar considerations motivated Cochrane and Piazzesi (2008), among others, to enforce even more persistent unit-root behavior under \mathbb{P} in their models.

²¹Alternative approaches to addressing small-sample bias in the estimates of \mathbb{P} distribution in dynamic term structure models include the near-cointegration analysis of Jardet, Monfort, and Pegoraro (2011) and the bootstrap methods used by Bauer, Rudebusch, and Wu (2011).

(AIC), Hannan and Quinn (1979) (HQIC), and Schwarz (1978) Bayesian information (SBIC) criteria.²² The criteria HQIC and SBIC are consistent (i.e., asymptotically they select the correct configuration of zero restrictions on $[\Lambda_0 \ \Lambda_1]$), while the AIC criterion may asymptotically over-fit (have too few zero restrictions) with positive probability.²³

The model selected by both the HQIC and SBIC criteria has twelve restrictions: eleven zero restrictions on $[\Lambda_0 \ \Lambda_1]$ and the eigenvalue constraint (see [Appendix E](#) for further details). The AIC criterion calls for fewer zero restrictions. All three criteria call for enforcing near-cointegration through the eigenvalue constraint. We proceed to investigate the more parsimonious *MTSM* that enforces the eigenvalue and eleven zero restrictions on the market prices of the risks $\Lambda_{\mathcal{P}t}$ identified by the HQIC and SBIC criteria. We denote this *MTSM* with unspanned macro risks by \mathcal{M}_{us} .

5.2 Risk Premium Accounting: Model Comparison

Initially, we compare our preferred model \mathcal{M}_{us} to three other models: the unconstrained canonical model ($\mathcal{M}_{\text{us}}^{\text{nose1}}$); the model $\mathcal{M}_{\text{us}}^{\text{e}}$ obtained by imposing only the eigenvalue constraint; and model $\mathcal{M}_{\text{us}}^0$ which imposes the eleven zero restrictions on risk premiums through $[\Lambda_0 \ \Lambda_1]$, but not the eigenvalue constraint. *ML* estimates of the parameters governing the \mathbb{Q} distribution of Z_t from model \mathcal{M}_{us} are displayed in the first column of [Table 1](#).²⁴ The estimates for the other three models are virtually indistinguishable from these estimates, typically differing in the fourth decimal place. This says that the parameters of the \mathbb{Q} distribution are determined largely by the cross-sectional restrictions on bond yields, and not by their time-series properties under the \mathbb{P} distribution. Models $\mathcal{M}_{\text{us}}^{\text{e}}$ and \mathcal{M}_{us} exploit this precision to restrict the degree of persistence of Z_t under \mathbb{P} .

Thus, any differences in the model-implied risk premiums must be attributable to differences in either the model-implied loadings of the yields onto the pricing factors \mathcal{P}_t in [\(9\)](#), or the the feedback matrices $K_{\frac{\mathbb{P}}{Z}}$ (differences in the \mathbb{P} distributions of \mathcal{P}_t). The loadings are fully determined by the \mathbb{Q} parameters $(r_{\infty}^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma_{\mathcal{P}\mathcal{P}})$ ([Appendix A](#)).

²²Bauer (2011) proposes a complementary approach to model selection based on the posterior odds ratio from Bayesian analysis. Another potential approach to deal with over-parameterization is given in Duffee (2010). He places restrictions on the maximal Sharpe ratio. However, in our formulations with unspanned macro risks, the maximal Sharpe ratios are reasonable and such constraints would be slack; further a spanning model would not allow unspanned macro risks.

²³These properties apply both when the true process is stationary and when it contains unit roots, as is discussed in Lutkepohl (2005), especially Propositions 4.2 and 8.1.

²⁴Throughout our analysis asymptotic standard errors are computed by numerical approximation to the Hessian and using the delta method.

Param	\mathcal{M}_{us}	Param	$\mathcal{M}_{\text{us}}^{\text{nosel}}$	$\mathcal{M}_{\text{us}}^0$	$\mathcal{M}_{\text{us}}^e$	\mathcal{M}_{us}
$r_{\infty}^{\mathbb{Q}}$	0.0918 (0.0058)	$ \lambda_1^{\mathbb{P}} $	0.9838 (0.0096)	0.9939 (0.0089)	0.9972 (0.0005)	0.9971 (0.0005)
$\lambda_1^{\mathbb{Q}}$	0.9971 (0.0005)	$ \lambda_2^{\mathbb{P}} $	0.9412 (0.0221)	0.9541 (0.0112)	0.9408 (0.0222)	0.9539 (0.0111)
$\lambda_2^{\mathbb{Q}}$	0.9650 (0.0026)	$ \lambda_3^{\mathbb{P}} $	0.9412 (0.0221)	0.9541 (0.0112)	0.9408 (0.0222)	0.9539 (0.0111)
$\lambda_3^{\mathbb{Q}}$	0.8868 (0.0122)	$ \lambda_4^{\mathbb{P}} $	0.9313 (0.0333)	0.8867 (0.0452)	0.9311 (0.0333)	0.8819 (0.0513)
		$ \lambda_5^{\mathbb{P}} $	0.7633 (0.0433)	0.8606 (0.0267)	0.7631 (0.0432)	0.8625 (0.0349)

Table 1: ML estimates of the \mathbb{Q} parameters for our preferred model with unspanned macro risks \mathcal{M}_{us} : the long run mean of the short rate under \mathbb{Q} , $r_{\infty}^{\mathbb{Q}}$, and the eigenvalues of the feedback under \mathbb{Q} , $\lambda^{\mathbb{Q}}$, which control the \mathbb{Q} -rates of mean reversion of the factors. Also tabulated are the moduli of the eigenvalues of $K_Z^{\mathbb{P}}$ for models $\mathcal{M}_{\text{us}}^{\text{nosel}}$ (no model selection imposed), $\mathcal{M}_{\text{us}}^0$ (only risk premium zero constraints), $\mathcal{M}_{\text{us}}^e$ (only eigenvalue constraint), and \mathcal{M}_{us} (our preferred model), which determine the \mathbb{P} -rate of mean reversion. Asymptotic standard errors are given in parentheses.

We have just seen that the parameters ($r_{\infty}^{\mathbb{Q}}$, $\lambda^{\mathbb{Q}}$) are nearly identical across models and, as it turns out, so are the ML estimates of $\Sigma_{\mathcal{P}\mathcal{P}}$. Consequently, the loadings (A_m, B_m) are also (essentially) indistinguishable across the four models examined.

In contrast, there are notable differences in the estimated feedback matrices $K_Z^{\mathbb{P}}$. The eigenvalues of $K_Z^{\mathbb{P}}$ (columns four through seven of [Table 1](#)),²⁵ reveal that the largest \mathbb{P} -eigenvalue in the canonical model $\mathcal{M}_{\text{us}}^{\text{nosel}}$ is smaller than in the constrained models. Its small value implies that expected future short-term rates beyond ten years are (nearly) constant or, equivalently and counterfactually, that virtually all of the variation in long-dated forward rates arises from variation in risk premiums.

Comparing across models also sheds light on the effects of our constraints on the \mathbb{P} -persistence of the risk factors. Enforcing the eleven zero restrictions in model $\mathcal{M}_{\text{us}}^0$ increases the largest eigenvalue of $K_Z^{\mathbb{P}}$ from 0.984 to 0.994, and thus closes most of the gap between models $\mathcal{M}_{\text{us}}^{\text{nosel}}$ and \mathcal{M}_{us} . In model $\mathcal{M}_{\text{us}}^0$, Z_t is sufficiently persistent under \mathbb{P} for long-dated forecasts of the short rate to display considerable

²⁵The fact that there are pairs of equal moduli in all three models means that there are complex roots in $K_Z^{\mathbb{P}}$. The complex parts were small in absolute value.

\mathcal{P}	const	$PC1$	$PC2$	$PC3$	GRO	INF
$PC1$	0	-0.0896 (0.0157)	-0.0510 (0.0122)	0	0.1083 (0.0313)	0.1729 (0.0326)
$PC2$	0	0	0	-0.1035 (0.0330)	-0.1487 (0.0307)	0.0486 (0.0123)
$PC3$	0	0	0	0	0	0

Table 2: ML estimates from our preferred model with unspanned macro risks, \mathcal{M}_{us} of the parameters Λ_0 and Λ_1 governing expected excess returns on the PC -mimicking portfolios: $xPC = \Lambda_0 + \Lambda_1 Z_t$. Standard errors are given in parentheses. Zeros are from our model selection.

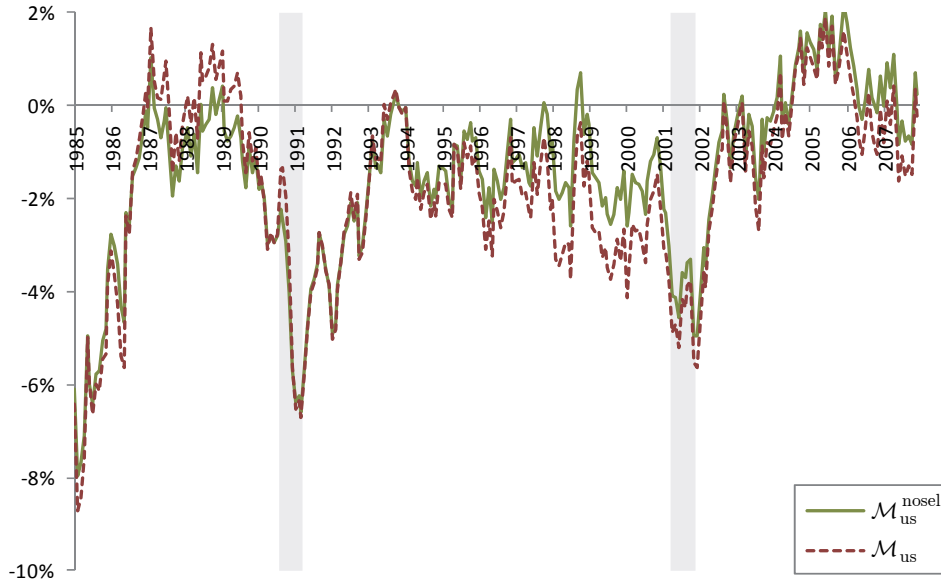
time variation. A further increase in the largest eigenvalue of $K_{\mathbb{Z}}^{\mathbb{P}}$ comes from adding the eigenvalue constraint in model \mathcal{M}_{us} .

Estimates from model \mathcal{M}_{us} of the parameters governing the expected excess returns $xPCj_t$ ($j = 1, 2, 3$) are displayed in [Table 2](#). The first and second rows of Λ_1 have non-zero entries, while the last row is set to zero by our model selection criteria. It follows that exposures to $PC1$ and $PC2$ risks are priced, but exposure to $PC3$ risk is not priced, at the one month horizon and during our sample period. That both level and slope risks are priced, instead of just level risk as presumed by Cochrane and Piazzesi (2008), is one manifestation of the important influence of macro factors on risk premiums.²⁶ GRO and INF both have statistically significant effects on $xPC1$ and $xPC2$. $xPC1$ is also influenced by $PC1$ and $PC2$, while $xPC2$ also depends on the curvature factor $PC3$.

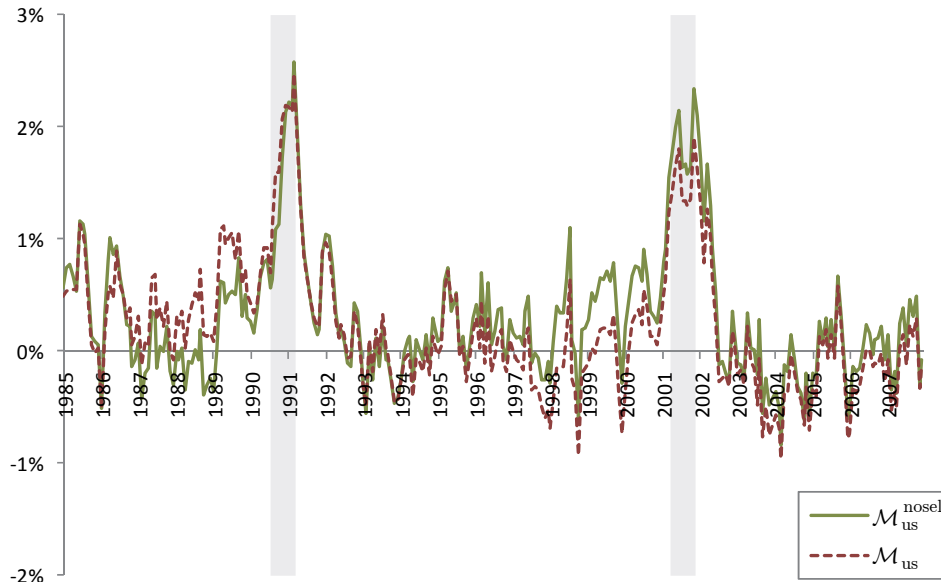
The signs of the coefficients imply that shocks to GRO induce pro- (counter-) cyclical movements in the risk premiums associated with exposures to $PC1$ ($PC2$). These effects can be seen graphically in [Figure 3](#) for models \mathcal{M}_{us}^{nose1} and \mathcal{M}_{us} , where the shaded areas represent the NBER-designated recessions. Exposures to $PC1$ ($PC2$) lose money when rates fall (the curve flattens), which is when investors holding long level (slope) positions make money. This explains the predominantly negative (positive) expected excess returns on the annualized $xPC1$ ($xPC2$), and why it is small (large) during the 1990 and 2001 recessions. There is broad agreement on the fitted excess returns across models \mathcal{M}_{us}^{nose1} and \mathcal{M}_{us} .

The premium on $PC2$ risk achieves its lowest value, and concurrently the premium on $PC1$ risk achieves its highest value, during 2004/05. Between June, 2004 and

²⁶With a model fit to yields alone, Duffee (2010) also finds evidence for two priced risks.



(a) Excess Return on Level-Mimicking Portfolio



(b) Excess Return on Slope-Mimicking Portfolio

Figure 3: Expected excess returns on the level- and the slope-mimicking portfolios implied by our preferred model with unspanned macro risks, \mathcal{M}_{us} , and the counterpart without model selection applied, \mathcal{M}_{us}^{nose1} .

Z	$K_0^{\mathbb{P}}$	$K_Z^{\mathbb{P}}$				
		$PC1$	$PC2$	$PC3$	GRO	INF
$PC1$	0.0002 (0.0000)	0.9138 (0.0156)	-0.0211 (0.0121)	-0.0482 (0.0031)	0.1083 (0.0313)	0.1729 (0.0326)
$PC2$	-0.0004 (0.0001)	-0.0188 (0.0012)	0.9697 (0.0017)	0.0426 (0.0327)	-0.1487 (0.0307)	0.0486 (0.0123)
$PC3$	0.0007 (0.0001)	0.0155 (0.0016)	0.0010 (0.0023)	0.8757 (0.0117)	0	0
GRO	0.0009 (0.0006)	-0.0157 (0.0144)	0.0191 (0.0109)	-0.1035 (0.0381)	0.8889 (0.0262)	0.0233 (0.0322)
INF	0.0003 (0.0002)	-0.0002 (0.0086)	0.0090 (0.0056)	-0.0395 (0.0223)	0.0347 (0.0161)	0.9966 (0.0194)

Table 3: Maximum Likelihood Estimates of $K_0^{\mathbb{P}}$ and $K_Z^{\mathbb{P}}$ for our preferred model with unspanned macro risks (\mathcal{M}_{us}): $E_t^{\mathbb{P}}[Z_{t+1}] = K_0^{\mathbb{P}} + K_Z^{\mathbb{P}}Z_t$. Standard errors are reported in parentheses.

June, 2006 the Federal Reserve increased its target Federal Funds rate by 4% (from 1.25% to 5.25%). Yields on ten-year Treasuries actually fell during this time, leading to a pronounced flattening of the yield curve, what Chairman Greenspan referred to as a conundrum. We revisit these patterns subsequently.

ML estimates of $K_0^{\mathbb{P}}$ and $K_Z^{\mathbb{P}}$ governing the \mathbb{P} -drift of Z_t are displayed in [Table 3](#) for model \mathcal{M}_{us} .²⁷ The non-zero coefficients on (GRO_{t-1}, INF_{t-1}) in the rows for $(PC1, PC2)$ are all statistically different from zero at conventional significance levels, confirming that macro information is *incrementally* useful for forecasting future bond yields after conditioning on $\{PC1, PC2, PC3\}$. Additionally, the coefficients on the own lags of GRO and INF are large and significantly different from zero, as expected given the high degree of persistence in these series.

For comparison, we also estimate a model $\mathcal{M}_{\text{span}}$ that enforces spanning of the forecasts of output growth and expected inflation by the yield PC s. Recall this is the nested special case with the last two columns of $K_Z^{\mathbb{P}}$ set to zero, as in [\(12\)](#).

²⁷The zeros in row $PC3$ follow from the zero constraints on Λ_1 . A zero in Λ_1 means that the associated factor has the same effect on the \mathbb{P} -forecasts as \mathbb{Q} -forecasts (i.e., $K_{\mathcal{P}\mathcal{P},ij}^{\mathbb{Q}} = K_{\mathcal{P}\mathcal{P},ij}^{\mathbb{P}}$). Since, by construction, the macro factors do not incrementally affect the \mathbb{Q} -expectations of the PC s, it follows that M_t has no effect on the \mathbb{P} -forecasts of $PC3$.

Similar models with macro spanning, based on the analyses of Bernanke, Reinhart, and Sack (2004) and Kim and Wright (2005), are referenced by Chairman Bernanke in discussions of the impact of the macro economy on bond risk premiums. For our choices of macro factors (GRO, INF), the χ^2 statistic for testing the null hypothesis that the last two columns of $K_Z^{\mathbb{P}}$ are zero is 1,189 (the 5% cutoff is 18.31). As we next show, the misspecified model $\mathcal{M}_{\text{span}}$ implies very different term premiums than model \mathcal{M}_{us} with unspanned macro risks.

6 Forward Term Premiums

Excess holding period returns on portfolios of individual bonds reflect the risk premiums for every segment of the yield curve up to the maturity of the underlying bond. A different perspective on market risk premiums comes from inspection of the forward term premiums, the differences between forward rates for a q -period loan to be initiated in p periods and the expected yield on a q -period bond purchased p periods from now. Within affine $MTSMs$, both forward rates and expected future q -year rates (and thus their difference) are affine functions of the state Z_t : $FTP_t^{p,q} = f_0^{p,q} + f_Z^{p,q} \cdot Z_t$.

To illustrate the differences between the risk premiums implied by $MTSMs$ with and without macro spanning, we display in [Figure 1](#) three different variants of the “in-two-for-one” forward term premium $FTP^{2,1}$. One is the fitted premium from our selected model \mathcal{M}_{us} with unspanned macro risks. The projection of this premium onto \mathcal{P}_t is displayed as PM_{us} . By construction, the \mathcal{M}_{us} premium depends on the entire set of risk factors Z_t , and any differences between \mathcal{M}_{us} and PM_{us} arise entirely from the effect of the unspanned components of M_t on $FTP_t^{2,1}$. The \mathcal{M}_{us} premium shows pronounced counter-cyclical swings about a gently downward-drifting level. The differences between the \mathcal{M}_{us} and PM_{us} premiums induced by unspanned macro risks are largest during the late 1980’s and the conundrum period, as well as at most peaks and troughs of $FTP^{2,1}$. These peak/trough differences are a consequence in large part of the dependence of the \mathcal{M}_{us} premium on GRO .

Equally striking from [Figure 1](#) are the very different patterns in the fitted $FTP^{2,1}$ from model \mathcal{M}_{us} and the premium from model $\mathcal{M}_{\text{span}}$ that constrains $E_{t-1}[M_t]$ to be spanned by \mathcal{P}_{t-1} (as in [\(12\)](#)). Both PM_{us} and $\mathcal{M}_{\text{span}}$ are graphs of premiums that are spanned by \mathcal{P}_t . However, they will coincide only when the macro-spanning constraint imposed in model $\mathcal{M}_{\text{span}}$ is consistent with the data-generating process for Z_t . In fact, the cyclical turning points of the premiums from models \mathcal{M}_{us} and $\mathcal{M}_{\text{span}}$ are far from synchronized, $\mathcal{M}_{\text{span}}$ drifts much lower during the late 1990’s, and it stays (relatively) high after the burst of the dotcom bubble when \mathcal{M}_{us} was declining along with the Federal Reserve’s target federal funds rate. Clearly the macro-spanning

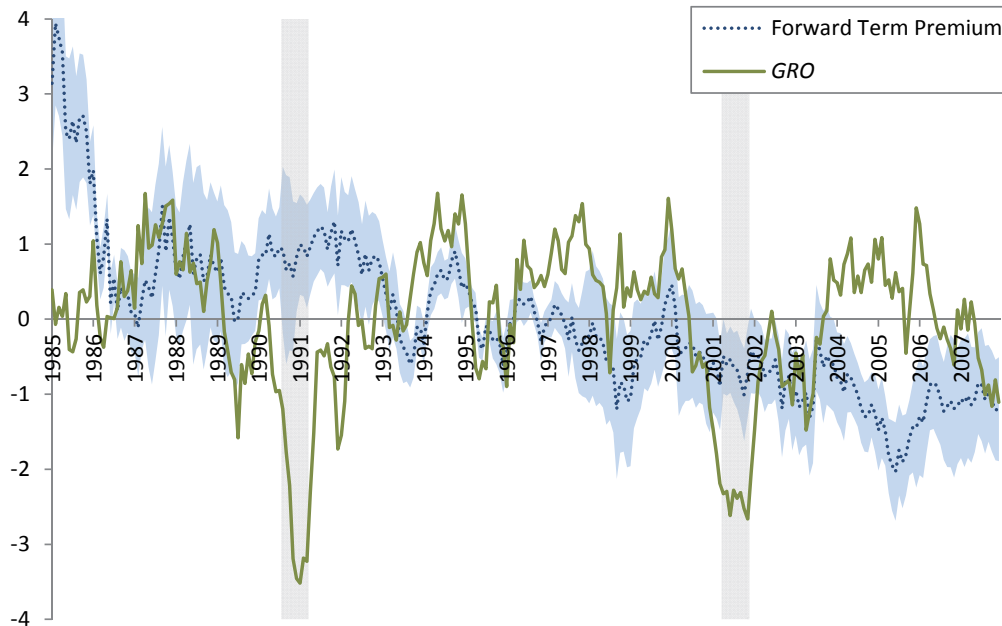


Figure 4: Standardized “in-9-for-1” forward premium from our preferred model with unspanned macro risk, \mathcal{M}_{us} , plotted against standardized GRO . The shaded band around $FTP^{9,1}$ is the 95% confidence band. $FTP^{9,1}$ is defined as the difference between the expectation for nine years in the future of the one-year yield and the forward rate that one could lock in today for a one-year loan commencing in nine years. GRO is the Chicago Fed National Activity Index.

constraint distorts the fitted risk premiums in economically significant ways.

Turning to longer-dated forward term premiums, the standardized “in-nine-for-one” premium $FTP^{9,1}$ is displayed in Figure 4, along with a standardized version of GRO . The band about the fitted $FTP^{9,1}$ is the 95% confidence band based on the precision of the ML estimates of $f_Z^{9,1}$. Importantly, with conditioning on both the macro factors and the shape of the yield curve, the implied $FTP^{9,1}$ does not follow an unambiguously counter-cyclical pattern. $FTP^{9,1}$ is high during the recession of the early 1990’s. However, during 1993 through 2000, there are subperiods when GRO and $FTP^{9,1}$ track each other quite closely.

The sources of this pro-cyclicity are revealed by the estimated coefficients $f_Z^{p,1}$ that link the FTP s to Z_t (Table 4). The negative weights on GRO and INF induce counter-cyclical movements in FTP s.²⁸ However, all three PC s have statistically

²⁸Complementary evidence that real economic activity affects expected excess returns on short-dated federal funds futures positions is presented in Piazzesi and Swanson (2008).

	const	<i>PC1</i>	<i>PC2</i>	<i>PC3</i>	<i>GRO</i>	<i>INF</i>
2-for-1	-0.0153 (0.0108)	1.1522 (0.1987)	0.2227 (0.1841)	0.9066 (0.3768)	-0.5950 (0.2786)	-1.5141 (0.4280)
5-for-1	-0.0103 (0.0137)	0.9090 (0.1166)	0.4179 (0.1238)	0.4262 (0.2641)	0.2430 (0.1737)	-0.8301 (0.3989)
9-for-1	-0.0023 (0.0151)	0.7392 (0.0813)	0.5341 (0.0983)	0.6875 (0.1600)	0.0828 (0.1046)	-0.7623 (0.2994)

Table 4: Coefficients $f_0^{p,1}$ and $f_Z^{p,1}$ determining the mapping between the forward term premiums $FTP_t^{p,1}$ and the state Z_t in our preferred model with unspanned macro risks, \mathcal{M}_{us} : $FTP_t^{p,1} = f_0^{p,1} + f_Z^{p,1} \cdot Z_t$. $FTP^{p,1}$ is defined as the difference between the expectation for p years in the future of the one-year yield and the forward rate that one could lock in today for a one-year loan commencing in p years.

significant, positive effects on $FTP^{9,1}$. *PC1* in particular followed a pro-cyclical path during the 1990's (Figure 2), and the FTP s reflect a blending of the influences of the priced level and slope risks. Evidently, there were important economic forces driving term premiums that were orthogonal to output growth and inflation.

We turn next to a more in-depth exploration of the contributions of unspanned macro risks to variation in risk premiums.

7 Spanned and Unspanned Macro Risks

An intriguing aspect of our empirical findings is the horizon-dependence of the impact of macro risk factors on risk premiums in the Treasury market. Figure 3 shows distinct cyclical patterns for level and slope risk premiums. Additionally, the loadings in Table 4 imply that the effect of *GRO* on the $FTP^{p,1}$ decline markedly, while those for *INF* remain large and of the same sign, as the contract horizon p increases. To what extent are the cyclical risk profiles of Treasury bonds determined by shocks to unspanned versus spanned macroeconomic factors?

Analogous to the loadings on *GRO* in Table 4 we find that a positive innovation to *GRO* tends to lower $FTP^{1,1}$, while (as the table shows) being largely neutral for $FTP^{9,1}$, thus inducing a steepening of the forward premium curve (increase in $SLF_1^9 \equiv FTP^{9,1} - FTP^{1,1}$). The impulse responses (*IRs*) of SLF_1^9 to innovations in spanned (*SGRO*) and unspanned (*OGRO*) output growth are displayed in Figure 5a

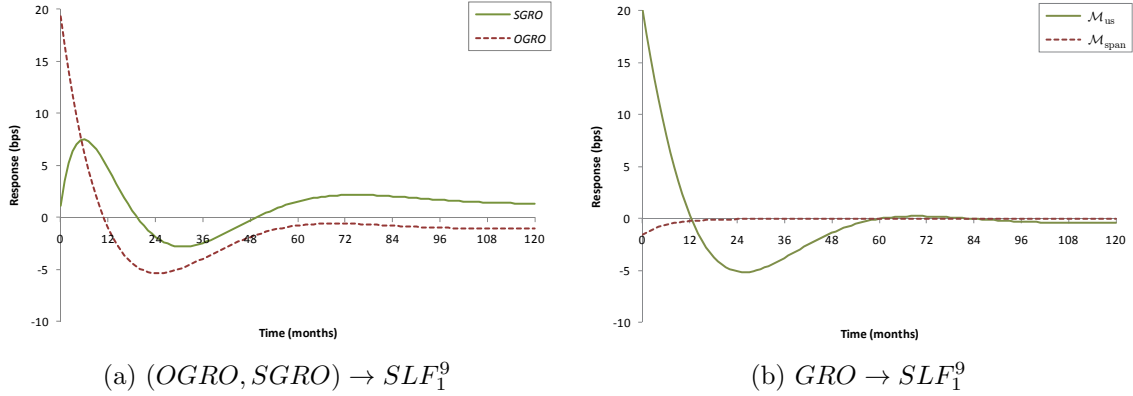


Figure 5: The left panel plots the impulse responses of the slope of the forward premium curve, $SLF_1^9 = FTP^{9,1} - FTP^{1,1}$, to shocks to either $(SGRO, OGRO)$ from our preferred model with unspanned macro risks \mathcal{M}_{us} . The right panel compares the responses of SLF_1^9 to innovations in total GRO across our preferred model with unspanned macro risks, \mathcal{M}_{us} , and the nested model that enforces spanning of expectations of the macro variables by the yield PCs (\mathcal{M}_{span}). $SGRO$, spanned growth, is the projection of GRO onto the PCs ; $OGRO$ is the component of GRO orthogonal to the PCs . $FTP^{p,1}$ is defined as the difference between the expectation for p years in the future of the one-year yield and the forward rate that one could lock in today for a one-year loan commencing in p years.

for model \mathcal{M}_{us} .²⁹ A shock to $OGRO$ induces an immediate, large steepening of the forward premium curve, and its effect then dissipates rapidly over the following year. This dominant role for $OGRO$ emerges even though $SGRO$ is ordered first in the underlying VAR . The macro-spanning restriction rules out any effect of $OGRO$ on SLF_1^9 .

Moreover, macro-spanning restrictions severely distort the responses of SLF_1^9 to shocks to total output growth GRO . The response of SLF_1^9 to a GRO shock in model \mathcal{M}_{us} (Figure 5b) looks nearly identical to its response to $OGRO$ in the adjacent figure, a manifestation of the large unspanned component of GRO . On the other hand, under macro spanning in model \mathcal{M}_{span} , the response of SLF_1^9 to a growth shock is (virtually) zero at all horizons. Evidently, shutting down the feedback from GRO to future Z drives out the economically important effects of growth on the

²⁹These IRs are computed from the (ordered) VAR of $(SGRO, SINP, OGRO, OINF, SLF_1^9)$ implied by model \mathcal{M}_{us} , where $SGRO$ is the model-implied projection of GRO_t onto the PCs \mathcal{P}_t and $OGRO_t$ is the residual from this projection.

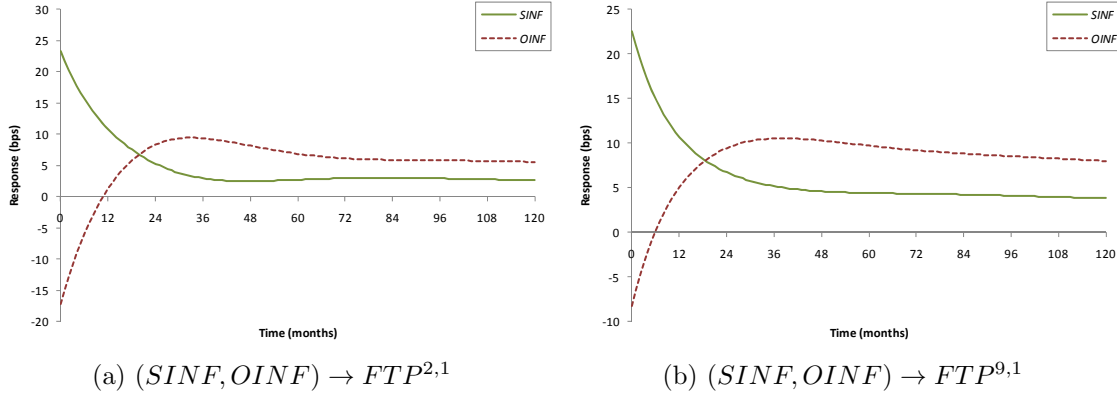


Figure 6: Each panel plots the impulse responses of FTP s to shocks to spanned and unspanned inflation, $(SINF, OINF)$ implied by our preferred model with unspanned macro risks, \mathcal{M}_{us} . $SINF$ is the projection of INF onto the PC s; $OINF$ is the component of INF orthogonal to the PC s. $FTP^{p,1}$ is defined as the difference between the expectation for p years in the future of the one-year yield and the forward rate that one could lock in today for a one-year loan commencing in p years.

slope of the forward premium curve.

In comparison to GRO , survey expectations of inflation are largely spanned by \mathcal{P}_t (85% of its variation) and INF shows higher persistence. The latter property of INF gives it a level-like effect in that innovations in INF (roughly uniformly) affect the entire maturity spectrum of yields. While the former property might lead one to presume that shocks to unspanned inflation ($OINF$) have inconsequential effects on risk premiums, this is not the case. These properties of inflation risk can be seen from the IRs of $FTP_t^{p,1}$, $p = 2, 9$, to shocks to $OINF$ and $SINF$ displayed in [Figure 6](#). The effects of $OINF$ persist for several years, owing to the near cointegration of INF with the priced risk factors ($PC1, PC2$).

There is also near symmetry in the effects of $(SINF, OINF)$ on forward term premiums. Initially, unspanned inflation shocks lead to lower FTP s, but the effects turn positive within a year. Innovations in $SINF$, in contrast, have large positive impact effects on FTP s that dissipate slowly over a couple of years. The dominant effect on $FTP^{9,1}$ at long horizons comes from unspanned inflation risk.

Returning to the period of the conundrum during 2004/05, notice from [Figure 4](#) that this was a period when GRO was increasing and long-dated forward term premiums were falling. In speaking about the conundrum, Chairman Bernanke asserted that “a substantial portion of the decline in distant-horizon forward rates

of recent quarters can be attributed to a drop in term premiums. ... the decline in the premium since June 2004 appears to have been associated mainly with a drop in the compensation for bearing real interest rate risk.”³⁰ According to our model, the forward term premium $FTP^{9,1}$ indeed declined by 112 basis points between June 2004 and June 2005,³¹ but by June 2006 had retracted to almost exactly its June 2004 level.

As to whether these patterns reflect changing premiums on real interest rate risk, of the initial decline, about 30 basis points can be attributed to orthogonal inflation, almost none to orthogonal growth, with the remainder accounted for by factors spanned by yields. Complementary to this, we find that the expected excess return on a bond portfolio mimicking the negative of *spanned* inflation—an indicator of the compensation required by investors facing spanned inflation risk—fell by roughly 60 basis points between June 2004 and June 2005. Both findings are indicative of a potentially more significant role played by inflation risks during the conundrum period than suggested by Chairman Bernanke.

Symmetric to this discussion is the interesting question of how changes in term premiums affect real economic activity. Bernanke, in his 2006 speech, argues that a higher term premium will depress the portion of spending that depends on long-term interest rates and thereby will have a dampening economic impact. In linearized New Keynesian models in which output is determined by a forward-looking IS equation (such as the model of Bekaert, Cho, and Moreno (2010)), current output depends only on the expectation of future short rates, leaving no role for a term premium effect. Time-varying term premiums do arise in models that are linearized at least to the third order (e.g., Ravenna and Seppala (2008)). We examine the response of real economic activity and inflation expectations to innovations in $FTP^{p,1}$ ($p = 2, 9$) in the context of model \mathcal{M}_{us} .³²

Initially, a one standard deviation increase in $FTP^{2,1}$ has a small negative impact on *OGRO* over a period of about eighteen months, and has virtually no effect on *SGRO* (Figure 7a). Shocks to the long-term premium $FTP^{9,1}$ induce a short-lived *positive* effect on *unspanned GRO* (Figure 7b). Again the premium shock has no effect on *SGRO*.³³ These responses present a more differentiated perspective on the

³⁰See his speech before the Economic Club of New York on March 20, 2006 titled “Reflections on the Yield Curve and Monetary Policy.”

³¹And by an even larger 175 basis points between July 2003 and June 2005.

³²The ordering of the model-implied *VAR* is $(FTP^{p,1}, SGRO, SINP, OGRO, OINF)$.

³³The absence of effects on *SGRO* is consistent with the results in Ang, Piazzesi, and Wei (2006) that term premiums are insignificant in predicting future GDP growth within an *MTSM* that enforces spanning of *GDP* growth by bond yields. What their model does not accommodate is our finding that term premium shocks do affect growth through their effects on unspanned real activity.

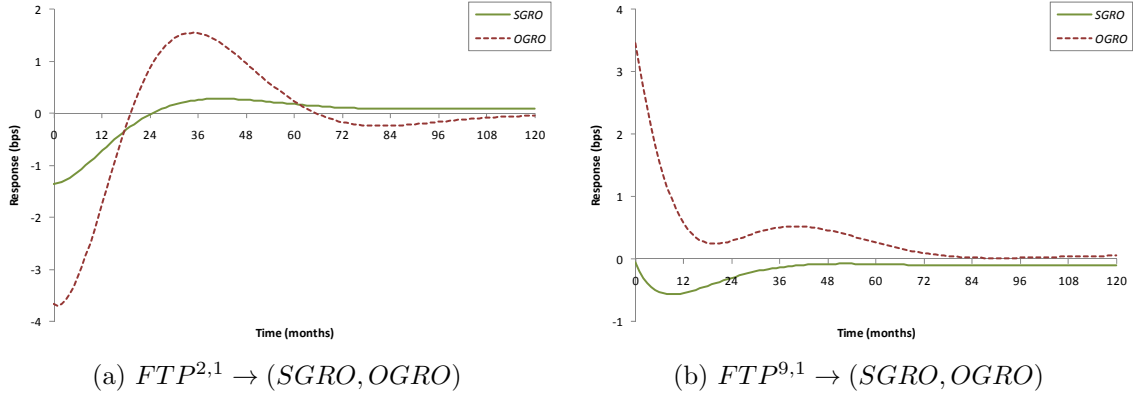


Figure 7: Each panel plots the impulse responses of spanned and unspanned growth, $(SGRO, OGRO)$ to shocks in FTP s implied by our preferred model with unspanned macro risks, \mathcal{M}_{us} . $SGRO$ is the projection of GRO onto the PC s; $OGRO$ is the component of GRO orthogonal to the PC s. $FTP^{p,1}$ is defined as the difference between the expectation for p years in the future of the one-year yield and the forward rate that one could lock in today for a one-year loan commencing in p years.

economic linkages set forth by Chairman Bernanke. A negative impact on economic activity arises from short- to medium-term risk premiums, not long-dated premiums. Moreover, the effects are virtually entirely through unspanned real economic activity, a component of growth that is absent from the models he cites in his analysis.

8 Extended Sample Analysis

In estimating our macro term structure models, we face a tradeoff between the potential small-sample bias arising from our selected sample period and biases that would arise from non-stationarity owing to structural breaks in a longer sample. The existing literature is divided on how this tradeoff is best resolved.³⁴ Based on the existing research, we argue in Section 5 that our sample period (1985–2007) is the longest recent sample that can reasonably be classified as a single regime (free from structural breaks). Small-sample concerns are mitigated somewhat by sampling at a

³⁴Ang and Piazzesi (2003) and Ang, Dong, and Piazzesi (2007) use relatively long, post-war samples starting in the early 1950's. Rudebusch and Wu (2008) start their sample in 1970 under the caveat of structural stability concerns. Smith and Taylor (2009) cite evidence of a structural break in the early 1980's and consequently proceed with a split-sample analysis around that break point.

monthly frequency, and by exploiting information about persistence from the pricing distribution. Though we document an economically important link between yields and unspanned macro risks, such links may well be different in different time periods. Indeed, it is our prior that unspanned macro risks are qualitatively important across policy regimes, but likely in quantitatively different ways.

8.1 Regime-Switching Model

To explore this conjecture jointly with our assumption that the post-1985 period is adequately treated as a single regime, we estimate a regime-switching version of our model, adapting the methodology of Dai, Singleton, and Yang (2007). Our extended sample starts in November 1971 (the earliest date with consistent availability of 10-year yield data, as discussed, for example, in Gurkanyak, Sack, and Wright (2007)) and December 2007. We use the same cross section of yields, and the same macroeconomic growth indicator (the Chicago Fed National Activity Index) as before, but we can no longer use Blue Chip inflation forecasts, as these are not available prior to the early 1980's. Instead, we define INF as the 12-month moving average of core CPI inflation, a measure that is highly correlated ($> 90\%$) with Blue Chip inflation forecasts over the period for which both inflation measures are available. We allow for two regimes with time homogeneous transition matrices $\pi^{\mathbb{P}}$ and $\pi^{\mathbb{Q}}$. All parameters except $\lambda^{\mathbb{Q}}$ are permitted to depend on the current regime (the maximally flexible regime-switching specification under which bond prices remain exponential-affine). We do not otherwise constrain parameters or perform a model-selection procedure as we did in Section 5.

Figure 8 shows that the maximum-likelihood-based regime classification³⁵ is indicative of a structural break in our data occurring in the mid-1980's, broadly consistent with the consensus in the literature. To a first approximation, the sample is divided into an early (pre-1985) and late (post-1985) regime. In particular, with the exception of the first year and three isolated months, the sample period we use in our main analysis is contained within a single regime. Conversely, the pre-1985 period is predominantly classified as a different regime. This finding supports our claim that the 1985–2007 period is adequately treated as a single regime, while this would not be the case for a longer sample.

The two regimes differ in economically meaningful ways, and consistent with the findings in previous research. In regime 2 (the “late” regime), the long-term mean of the short rate under the risk neutral measure is lower, the system of yields and

³⁵As is standard, we classify an observation as belonging to the regime with the highest ex post (smoothed) probability.

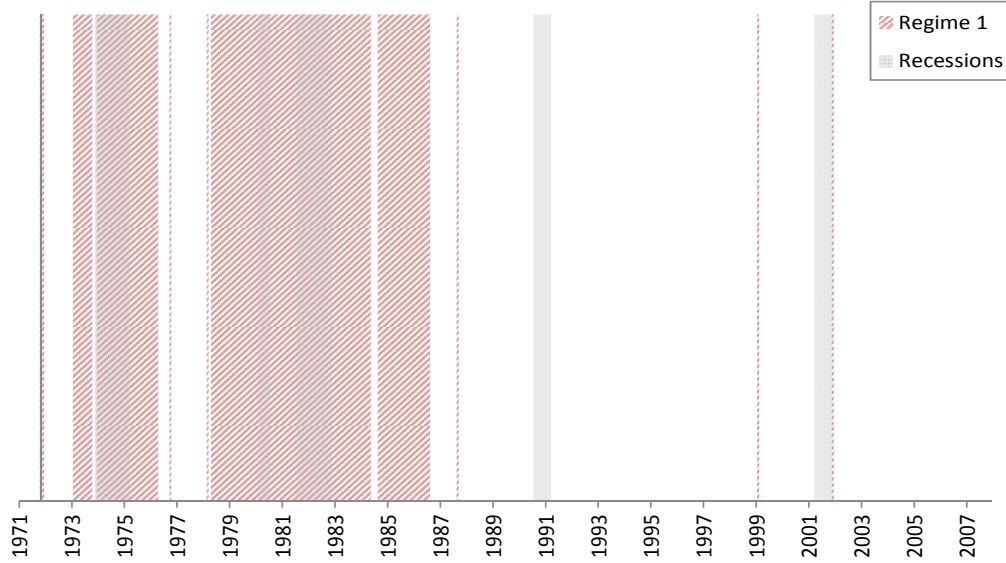


Figure 8: The figure shows the regime classification from a two-regime model with unspanned macro risks. The red hatched area represents the first regime, while shaded areas indicate NBER recession periods.

macro variables is more persistent and has lower conditional volatility (reflecting the “Great Moderation” analyzed by Stock and Watson (2002)), and the regime is more stable. While GRO affects level and slope risk premiums comparably across regimes, not surprisingly given the high inflation in the first regime, INF has a much larger effect on level risk in regime 1 than regime 2 (for exact parameter estimates, see [Appendix F](#)). These differentiated results would be obscured if we estimated a single-regime model over the longer sample period.

8.2 Out-of-Sample Analysis

Also of interest are the properties of our model’s implied risk premiums during the post-2007 crisis period. Up to this point we have excluded this period out of concerns not only about another structure break, but also about the ability of a Gaussian term structure model to adequately capture yield dynamics near the zero lower bound. With this cautionary observation in mind, we briefly examine the out-of-sample differences between our preferred model with unspanned macro variables, \mathcal{M}_{us} , and the alternative model with spanned macro factors, \mathcal{M}_{span} . For this purpose we compute fitted risk premiums (based on model estimates for the 1985–2007 sample)

starting from the end of our estimation sample and continuing through July 2012.

Figure 9 plots the in-two-for-one forward term premiums implied by models \mathcal{M}_{us} and $\mathcal{M}_{\text{span}}$. As we discussed in Section 6, these forward term premiums bear limited resemblance in sample. Out of sample, the differences are even more pronounced, particularly in the 2008–2010 period. The term premium implied by model \mathcal{M}_{us} initially increases sharply in 2008 with a rapidly deteriorating economic outlook. It rebounds later that year, and declines by a similar magnitude in late 2010/early 2011. The declines roughly coincide with the Federal Reserve’s first two quantitative easing programs (QE1 and QE2), a stated objective of which was to lower forward term premiums. The movements in the $\mathcal{M}_{\text{span}}$ -implied forward term premium are much more subdued, and harder to reconcile with economic events. For instance, the forward term premium *increases* around QE1.

9 Elaborations and Extensions

Our framework can be applied in any Gaussian pricing setting in which security prices or yields are affine functions of a set of pricing factors \mathcal{P}_t and risk premiums depend on a richer set of state variables that have predictive power for \mathcal{P}_t under the physical measure \mathbb{P} . Accordingly, it is well suited to addressing a variety of economic questions about risk premiums in bond and currency markets, as well as in equity markets when the latter pricing problems map into an affine pricing model (e.g., Bansal, Kiku, and Yaron (2012b)). Though neither the state variables nor the pricing factors exhibit time-varying volatility in the settings examined in this paper, our basic framework and its computational advantages are likely to extend to affine models with time-varying volatility. Incorporating time-varying volatility would allow for the possibility of volatility factors that are unspanned by bond prices or macro variables, thereby generalizing Collin-Dufresne and Goldstein (2002). As in our setting, such unspanned volatility factors may also drive expected returns (as in Joslin (2013b)). Exploration of this extension is deferred to future research.

A distinct, though complementary, question is: what are the structural economic underpinnings of the substantial effects of unspanned macro variables on risk premiums that we documented in our empirical analysis? At this juncture we comment briefly on the insights that our reduced-form model has revealed about the nature of risk premiums in the U.S. Treasury market, again leaving the task of developing a structural model with these features to future research.

Many of the extant structural models of term premiums in bond markets rule out by construction any link between unspanned macro risks and term premiums. This is trivially the case in the model of Bekaert, Cho, and Moreno (2010), because

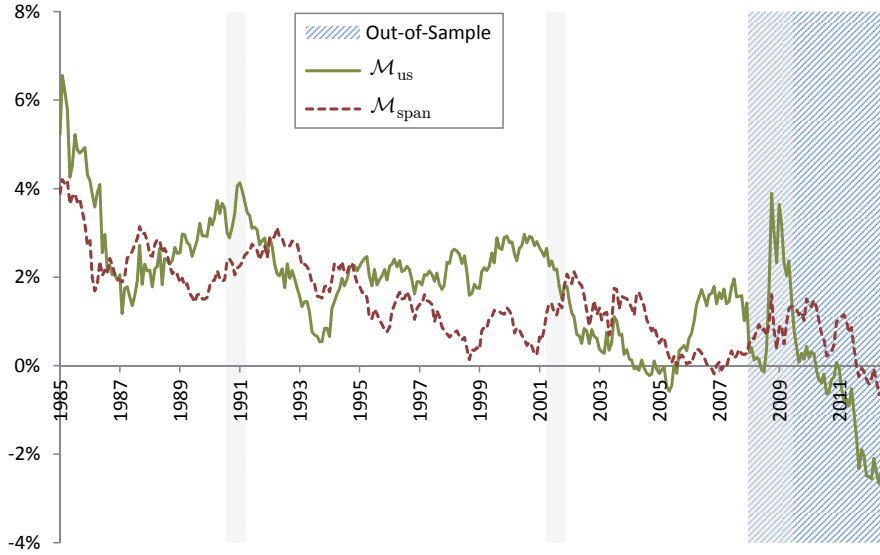


Figure 9: Out-of-sample forward term premiums $FTP^{2,1}$ for our preferred model with unspanned macro risks (\mathcal{M}_{us}), and the nested model that enforces spanning of expectations of the macro variables by the yield PCs (\mathcal{M}_{span}).

they assume constant risk premiums. Gallmeyer, Hollifield, Palomino, and Zin (2007) add preferences with habit shocks (as in Abel (1990)) to a policy rule to obtain a setting with time-varying risk premiums. However, their models also imply that real economic activity (in their case consumption growth) and inflation are fully spanned by the current yield curve. Indeed, all of the equilibrium models of bond yields that fall within the family of affine pricing models that we are aware of implicitly impose macro-spanning conditions (see Le and Singleton (2013)).

We can imagine how unspanned macro risks could arise through additional constraints on agents' preferences, policy rules, and exogenous sources of uncertainty. For instance, in a model with preferences of the habit type, if the short rate is affine in growth and one component of growth has the same mean reversion as the habit variable (for example, as growth decreases, risk aversion increases according to such a rule), then there may be unspanned growth risk. Within an affine pricing setting, such unspanned growth could be allowed to impact policy rules and the inflation process. Yet whether this avenue for accommodating unspanned risks is fruitful is ultimately a quantitative question that warrants further exploration. A primary take-away from our findings is that this, or some other complementary, economic mechanism seems needed to rationalize our core empirical findings.

Our findings also speak to the policy conclusions drawn from structural models. A standard framework for policy analysis within a structural term structure model is the New Keynesian model, as in Clarida, Gali, and Gertler (1999). For example, Gallmeyer, Hollifield, and Zin (2005) and Gallmeyer et al. (2007) examine affine term structure models that embed Taylor- or McCallum-style monetary policy rules and (implicitly) enforce spanning of all macro variables by bond yields. Our results suggest that a monetary authority may affect the output gap and inflation through channels that leave bond yields unaffected, by having a simultaneous effect on expectations about the future short rates and risk premiums.

10 Conclusion

This paper has explored the effects of unspanned macro risks on risk premiums in bond markets. We find that shocks to unspanned real economic activity and inflation have large effects on term premiums in U.S. Treasury markets and, symmetrically, shocks to forward term premiums have substantial effects on real economic activity primarily through their effects on unspanned real output growth. Moreover, we document an important role for unspanned inflation risks in shaping term premiums, despite the fact that a large portion of the variation in inflation is spanned by bonds.

In order to assess the role of unspanned macro risks, we develop a canonical arbitrage-free, Gaussian *MTSM* in which the state vector includes macroeconomic variables that are not perfectly spanned by contemporaneous bond yields, and in which these macro variables can have significant predictive content for excess returns on bonds over and above the information in bond yields. We show that this canonical representation of the model lends itself to easy interpretation and attains the global maximum of the likelihood function essentially instantaneously.

Properties of the fitted historical distributions of bonds and pricing factors in our *MTSM* are very different than what is implied by both a factor-*VAR* model or an unconstrained version of our canonical model. Owing to the constraints suggested by our model selection criteria, the persistence properties of bond yields, and hence the relative importance of expected future spot rates versus forward term premiums, are very different in our preferred model than in its unconstrained counterpart. This suggests that, when estimating *MTSMs*, one should undertake similar model-selection exercises to systematically reduce the dimension of the parameter space, as this might similarly mitigate small-sample bias problems.

Our findings raise several intriguing questions for future research. Unspanned macro risks, particularly real economic risks, had large effects on forward term premiums over short- to intermediate-term horizons. What were the economic sources

of these unspanned risks? On the other hand, we find that a substantial portion of the variation in long-dated forward term premiums is attributable to economic factors that are orthogonal to both spanned and unspanned output and inflation. It was evidently these orthogonal risks that largely explained the decline in term premiums during the period of the conundrum. What is the nature of these macro risks that are so important in Treasury markets and yet are orthogonal to output growth and inflation? Since the onset of the financial crisis, there has been considerable discussion about the roles of global imbalances and disruptions in the financial intermediation sectors. Our modeling framework provides a means of systematically examining these possibilities within arbitrage-free dynamic term structure models.

Appendices

A Bond Pricing in *GDTMs*

The price of an m -month zero-coupon bond is given by

$$D_{t,m} = E_t^{\mathbb{Q}}[e^{-\sum_{i=0}^{m-1} r_{t+i}}] = e^{\mathcal{A}_m + \mathcal{B}_m \cdot \mathcal{P}_t}, \quad (14)$$

where $(\mathcal{A}_m, \mathcal{B}_m)$ solve the first-order difference equations

$$\mathcal{A}_{m+1} - \mathcal{A}_m = (K_0^{\mathbb{Q}})' \mathcal{B}_m + \frac{1}{2} \mathcal{B}_m' \Sigma_{\mathcal{P}\mathcal{P}} \mathcal{B}_m - \rho_0 \quad (15)$$

$$\mathcal{B}_{m+1} - \mathcal{B}_m = (K_1^{\mathbb{Q}})' \mathcal{B}_m - \rho_1 \quad (16)$$

subject to the initial conditions $\mathcal{A}_0 = 0, \mathcal{B}_0 = 0$. See, for example, Dai and Singleton (2003). The loadings for the corresponding bond yield are $A_m = -\mathcal{A}_m/m$ and $B_m = -\mathcal{B}_m/m$.

B Conditions for Unspanned Risks

In this appendix, we derive the nominal pricing kernel (equivalently, risk neutral distribution of the N risk factors Z_t) in order to have a model with unspanned macro risks. We begin with the general specification the nominal pricing kernel in (1) with short rate given in (3) and dynamics of the state Z_t variable given by (2). Macro variables are added to the system through:

$$M_t = \gamma_{0Z} + \gamma_{1Z} \cdot Z_t. \quad (17)$$

Notice that this specification is the most general affine Gaussian model. In particular, it subsumes the case where some elements of Z_t are themselves macro variables by assuming that rows of γ_{1Z} are standard basis vectors (with corresponding entries in γ_{0Z} set to zero).

From the pricing kernel, we derive the risk-neutral distribution of Z_t through

$$\left. \frac{d\mathbb{Q}}{d\mathbb{P}} \right|_t = e^{-\sum_{s=1}^t [\frac{1}{2} \Lambda'_{Z,s} \Lambda_{Z,s} - \Lambda'_{Z,s} \eta_s^{\mathbb{P}}]}.$$

Our assumption is that the market prices of risk are affine so that, following the notation of Section 5, we have

$$\Lambda_{Z,t} = \Sigma_{ZZ}^{-1/2} (\Lambda_0 + \Lambda_1 Z_t)$$

and the risk-neutral distribution is given by

$$Z_t = K_{0Z}^{\mathbb{Q}} + K_{1Z}^{\mathbb{Q}} Z_{t-1} + \sqrt{\Sigma_Z} \eta_t^{\mathbb{P}} \eta_t^{\mathbb{P}} \sim N(0, I). \quad (18)$$

Here $(K_{0Z}^{\mathbb{Q}}, K_{1Z}^{\mathbb{Q}})$ are given through the market prices of risk:

$$K_{0Z}^{\mathbb{Q}} = K_{0Z}^{\mathbb{P}} - \Lambda_0 \quad (19)$$

$$K_{1Z}^{\mathbb{Q}} = K_{1Z}^{\mathbb{P}} - \Lambda_1 \quad (20)$$

Following Joslin (2013a), we see that the macro risks are unspanned macro risks if there are $(N - R)$ vectors $\{v_1, v_2, \dots, v_{N-R}\}$ so that for each i :

1. $v_i \cdot \rho_{1Z} = 0$.
2. $v_i' K_{1Z}^{\mathbb{Q}}$ is a multiple of $K_{1Z}^{\mathbb{Q}}$ (i.e., v_i is a left eigenvector of $K_{1Z}^{\mathbb{Q}}$.)
3. $\gamma_1 \cdot v_i \neq 0$

The first condition ensures that the factor $v_i \cdot Z_t$ does not effect the short rate while the second condition ensures that $v_i \cdot Z_t$ does not effect risk-neutral expectations of the short rate (for any horizon). The final condition ensures that the factor $v_i \cdot Z_t$ has an effect on the macro variables. We can relax this condition so that if $v_i \cdot Z_t = 0$ there can be a factor which is not identified by the joint cross section of bond prices and M_t .

Notice that the conditions above are all specified directly in term of the risk-neutral parameters. Alternatively, one could use (19) and (20) to express the conditions for unspanned macro risk in terms of the market prices of risk and \mathbb{P} -parameters.

C A Canonical Model with Unspanned Macro Risks

In this section we prove that our form of *MTSM* with unspanned macro risk is canonical in the following sense. Consider the family of *MTSM* driven by with nominal pricing kernel given in (1), with short rate given in (3), with dynamics of the state Z_t variable given by (2), and with macro variables given in (17). For Z_t of dimension N and M_t of dimension R , we denote $UMA_0^R(N)$ to be the family this family when the macro risks are unspanned. To prove that our family is canonical, we show that every such *MTSM* is observationally equivalent (given data on the macro variables and the cross section of all yields) to exactly one model with the specification given in Section 3.

To prove the existence of such a representation, we begin with a generic model with a latent Z_t driving yields and macro factors and apply the affine rotations as in Dai and Singleton (2000). Taking the loadings from (9) we may write

$$\begin{bmatrix} \mathcal{P}_t \\ M_t \end{bmatrix} = \begin{bmatrix} WA \\ \gamma_0 \end{bmatrix} + \begin{bmatrix} WB \\ \gamma_1 \end{bmatrix} Z_t, \quad (21)$$

where W are the weights in the principles components and (A, B) are the stacked loadings for the corresponding maturities. The matrix $[WB, \gamma_1]$ is invertible by the spanning assumptions in the previous section. Thus after we apply the affine transformation of (21) to our parameters, we obtain a model with (\mathcal{P}_t, M_t) as the state variable. The spanning assumptions in the previous section show that \mathcal{P}_t is \mathbb{Q} -Markov and that the short rate must depend only on \mathcal{P}_t (and not M_t). The results of Joslin, Singleton, and Zhu (2011) then allow us to uniquely parameterize the \mathbb{Q} -distribution of \mathcal{P}_t through $(\Sigma_{\mathcal{P}\mathcal{P}}, \lambda^{\mathbb{Q}}, r_{\infty}^{\mathbb{Q}})$. The \mathbb{Q} -distribution of M_t is not identified in our specification without further data. In the presence of securities with payoffs directly tied to M_t , the risk-neutral dynamics could be fully specified (equivalently, the entire pricing kernel rather than just the projection onto bond could be identified.)

D Returns on Generalized Mimicking Portfolios

Consider a collection of N yields, $\{y_t^{n_1}, \dots, y_t^{n_N}\}$, and a given linear combination $y_t^a = \sum_{i=1}^N a_i y_t^{n_i}$ of these yields (y_t^a could be a principal component, or the projection of a macro variable onto the yields). Our first goal is to find weights $\{w_i\}_{i=1}^N$ such that value $P_t^w = \sum_{i=1}^N w_i P_t^{n_i}$ of a portfolio of zero coupon bonds locally tracks changes in y_t^a ; that is,

$$\frac{dP_t^w}{dy_t^a} = \sum_{i=1}^N \frac{dP_t^w}{dy_t^{n_i}} \frac{dy_t^{n_i}}{dy_t^a} = 1 \quad (22)$$

Since, by definition, $P_t^{n_i} = \exp(-n_i y_t^{n_i})$, we have $dP_t^{n_i}/dy_t^{n_i} = -n_i P_t^{n_i}$. Therefore, (22) can be rewritten as

$$-\sum_{i=1}^N w_i n_i P_t^{n_i} \frac{1}{a_i} = 1$$

which will hold for weights

$$w_i = -\frac{a_i}{N n_i P_t^{n_i}}$$

Next, consider the one-period excess return on portfolio P_t^w :

$$\frac{\sum_i w_i (P_{t+1}^{n_i-1} - e^{r_t} P_t^{n_i})}{|\sum_i w_i P_t^{n_i}|} = \frac{-\sum_i a_i/n_i (P_{t+1}^{n_i-1}/P_t^{n_i} - e^{r_t})}{|\sum_i a_i/n_i|}.$$

This is a weighted average of the returns on the individual zero coupon bonds. Now, it follows from Le, Singleton, and Dai (2010) that $P_t^{n_i} = \exp(-A_{n_i} - B_{n_i} \mathcal{P}_t)$, and further that

$$E^{\mathbb{P}}[P_{t+1}^{n_i-1}/P_t^{n_i}] = \exp\{B_{n_i-1}[(K_0^{\mathbb{Q}} - K_0^{\mathbb{P}}) + (K_{\mathcal{P}}^{\mathbb{Q}} - K_{\mathcal{P}}^{\mathbb{P}})Z_t] + r_t\}.$$

Therefore, to a first-order approximation, the expected excess return on portfolio P_t^w is given by

$$\frac{\sum_i a_i/n_i B_{n_i-1}[(K_0^{\mathbb{P}} - K_0^{\mathbb{Q}}) + (K_{\mathcal{P}}^{\mathbb{P}} - K_{\mathcal{P}}^{\mathbb{Q}})Z_t]}{|\sum_i a_i/n_i|}.$$

Since we rotate our model such that the first R elements of Z_t correspond to the first R principal components of yields, and since by definition,

$$PCj_t = \sum_{i=1}^N \ell_i^j y_t^{n_i} = \sum_{i=1}^N \ell_i^j (A_{n_i}/n_i + B_{n_i}/n_i \mathcal{P}_t)$$

it follows that $\sum_i \ell_i^j B_{n_i}/n_i$ is the selection vector for the j^{th} element, $j \in \{1, \dots, R\}$. Thus, under the further approximation that $B_{n_i-1} \approx B_{n_i}$, the expected excess return on the portfolio mimicking PCj , $xPCj$, is given by the j^{th} row of

$$(K_0^{\mathbb{P}} - K_0^{\mathbb{Q}}) + (K_{\mathcal{P}}^{\mathbb{P}} - K_{\mathcal{P}}^{\mathbb{Q}})Z_t$$

scaled by $|\sum_i \ell_i^j/n_i|$. While an approximation for the one-period expected excess return in discrete time, this relationship is exact for the instantaneous expected excess return in the continuous-time limit.

E Details on Model Selection

As discussed in Section 5.1, we select among a total of 2^{19} model specifications distinguished by different sets of restrictions on the market price of risk parameters, and the maximum eigenvalue of $K_Z^{\mathbb{P}}$. In estimating the baseline (unrestricted) version of our model, we reduce the computational burden by taking advantage of the

likelihood function factorization (13). Given $(r_\infty^Q, \lambda^Q, L_Z)$, the remaining parameter estimates can be computed in closed form as the solution to a standard OLS problem. Thus, we only need to optimize numerically over $(r_\infty^Q, \lambda^Q, L_Z)$.

Estimators for the restricted versions of our models can be computed almost equally efficiently. First, given $(r_\infty^Q, \lambda^Q, L_Z)$, the zero restrictions on market price of risk parameters can be expressed as linear restrictions on (K_0^P, K_Z^P) . In the likelihood factorization (13), the first term is unaffected by these restrictions. Maximizing the second term with respect to (K_0^P, K_Z^P) amounts to estimating a restricted Seemingly Unrelated Regression problem, which has a known closed-form solution. Therefore, the computational complexity of the overall maximization problem is unchanged. Similarly, the eigenvalue constraint on K_Z^P can also be expressed as a linear restriction given λ^Q : The restriction can be written as $\det(K_Z^P - \lambda_{\max}^Q I) = 0 \Rightarrow (K_Z^P - \lambda_{\max}^Q I)b = 0$ for some vector b . Hence, this amounts to additional linear restrictions on K_Z^P , given b . However, the parameters in vector b must now be included in the set of parameters over which we optimize numerically. This makes the optimization problem somewhat more complex, although in practice we find convergence is still rapid.

The outcome of our model selection procedure can be summarized graphically as a “likelihood frontier,” as in Figure 10.

F Parameter Estimates for Regime-Switching Model

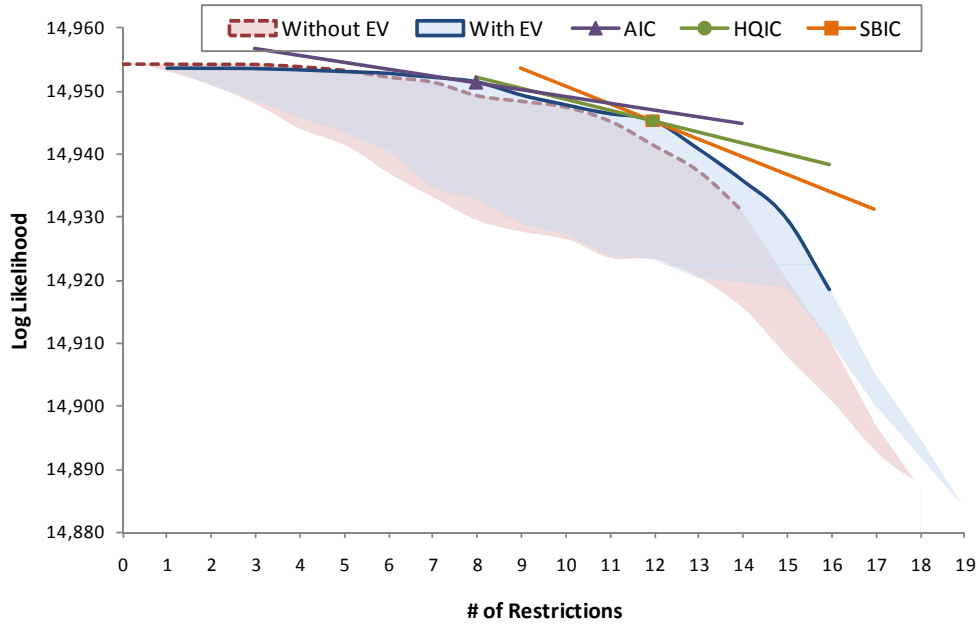


Figure 10: Each of the 2^{19} model specifications we consider corresponds to a point on the graph. The likelihood frontiers “Without EV” and “With EV” trace out the models with the highest likelihood for a given number of restrictions, without and with the maximum eigenvalue constraint on $K_{\frac{P}{Z}}$ imposed, respectively. The straight lines are “indifference curves” for each of the three information criteria. The tangency points correspond to the models selected under each criterion.

Param	Regime 1		Regime 2	
r_∞^Q	0.0889 (0.0196)		0.0773 (0.0189)	
$ \lambda_1^P $	0.9751 (0.0265)		0.9904 (0.0138)	
K_{PM}^P	0.0998 (0.0579)	0.0538 (0.0263)	0.1253 (0.0284)	0.0121 (0.0166)
	-0.0853 (0.0521)	-0.0303 (0.0237)	-0.1006 (0.0274)	0.0445 (0.0045)
	0.0822 (0.0401)	0.0004 (0.0182)	-0.0532 (0.0247)	0.0188 (0.0152)
$\sqrt{\text{diag}(\Sigma_Z)}$	0.0061		0.0025	
	0.0054		0.0024	
	0.0041		0.0021	
	0.0041		0.0020	
	0.0040		0.0014	

(a) Regime-dependent parameters

0.9301 0.0699

0.0352 0.9648

(b) Transition Probabilities π^P

Table 5: *ML* estimates of selected parameters for the regime-switching model based on an extended sample period 1971–2007. Asymptotic standard errors are given in parentheses.

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